Ministère de l'Enseignement Supérieur et de la Recherche Scientifique
Université Djilali Bounaama de Khemis Miliana
Faculté des Sciences et de la Technologie
Filière de Génie Civil

La méthode des Eléments Finis
De la théorie vers la pratique

Cours : 30-09-2020

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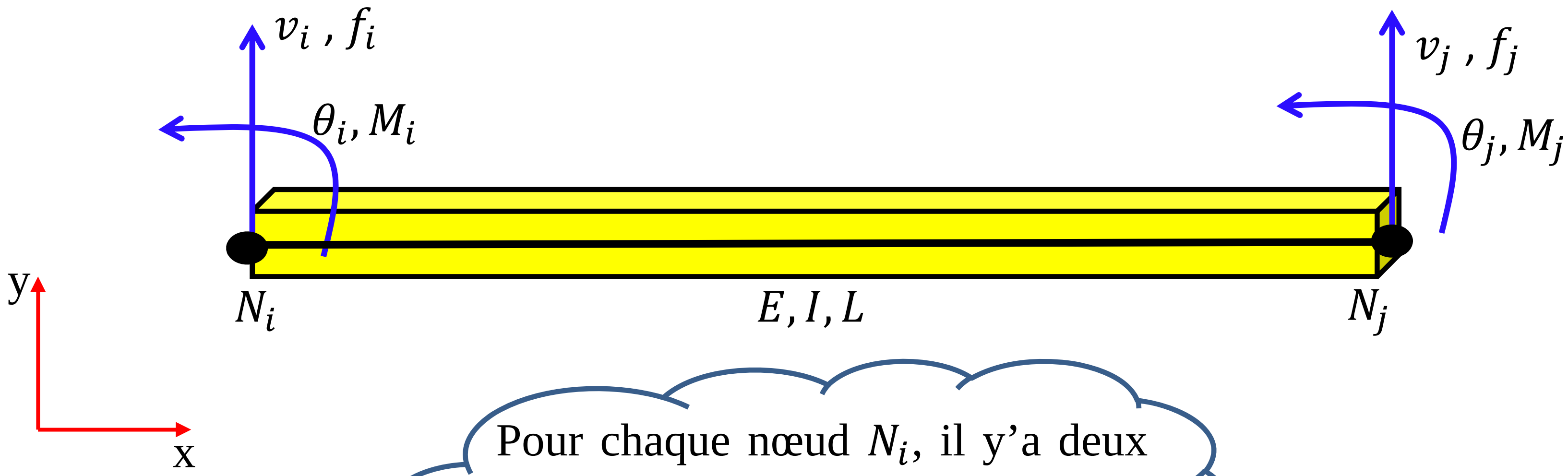
❖ Sens positif de l'application de la M.E.F poutre



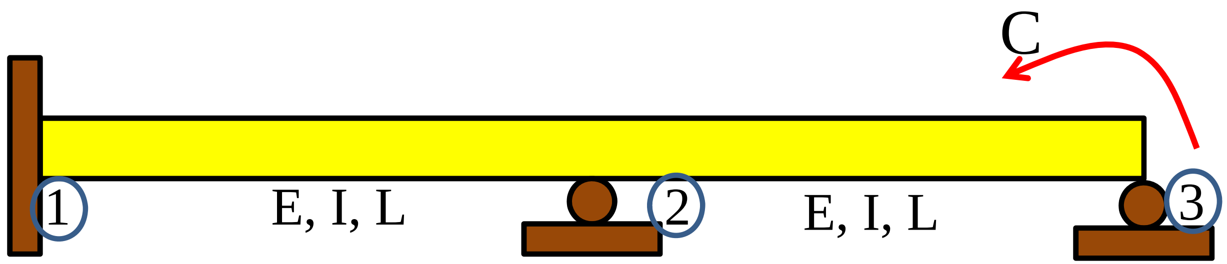
Pour chaque nœud N_i, il y'a deux variables (un déplacement v_i et une rotation θ_i)

✓ Implique que pour chaque nœud N_i, il y'a deux forces (une force f_i et un moment fléchissant M_i)

- ✓ La matrice de rigidité élémentaire pour l'élément finis poutre est donnée par :

$$[k_e] = \frac{EI_z}{L^3}\begin{array}{cc} \begin{array}{cc} \overbrace{\qquad\qquad\quad}^{N_i} & \overbrace{\qquad\qquad\quad}^{N_j} \end{array} & \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} & \begin{array}{c} \left.\begin{array}{c} \\ \\ \end{array}\right\} N_i \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} N_j \end{array} \end{array}$$

- Exemple



- Données

Elem	Ni	Nj	Ei	Ii	Li
I	1	2	E	I	L
II	2	3	E	I	L

- Matrice de rigidité élémentaire (4 x 4)

  - <u>Elément I: 1-2</u>

$$[k_I] = \frac{EI_z}{L^3}\begin{array}{cc} \begin{array}{cccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} \end{array} & \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} & \begin{array}{c} \left.\begin{array}{c} \\ \\ \end{array}\right\} 1 \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} 2 \end{array} \end{array}$$

  - <u>Elément I: 2-3</u>

$$[k_{II}] = \frac{EI_z}{L^3}\begin{array}{cc} \begin{array}{cccc} \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} \end{array} & \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} & \begin{array}{c} \left.\begin{array}{c} \\ \\ \end{array}\right\} 2 \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} 3 \end{array} \end{array}$$

- Matrice de rigidité globale ((2xnoued) x (2xnoued))

$$[K] = \frac{EI_z}{L^3}\begin{array}{c}\begin{array}{cccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} \end{array}\\ \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12 & -6L & 0 & 0 \\ 6L & 2L^2 & -6L & 4L^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\end{array}\begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ \end{array}$$

$$[K] = \frac{EI_z}{L^3}\begin{array}{c}\begin{array}{cccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} \end{array}\\ \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 12+12 & -6L+6L & -12 & 6L \\ 6L & 2L^2 & -6L+6L & 4L^2+4L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\end{array}\begin{array}{c} \\ 1 \\ \\ 2 \\ \\ 3 \\ \\ \end{array}$$

$$[K] = \frac{EI_z}{L^3} \begin{array}{cc} \begin{array}{cccccc} \multicolumn{2}{c}{1} & \multicolumn{2}{c}{2} & \multicolumn{2}{c}{3} \end{array} & \\ \begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & 0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} & \begin{array}{c} \left.\begin{array}{c} \\ \\ \end{array}\right\} 1 \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} 2 \\ \left.\begin{array}{c} \\ \\ \end{array}\right\} 3 \end{array} \end{array}$$

- *Conditions aux limites*

❖ Nœud 01: Encastrement ⇨ $v_1 = 0 \; et \; \theta_1 = 0$

❖ Nœud 02: Appui double ⇨ $v_2 = 0$

❖ Nœud 03: Appui double ⇨ $v_3 = 0$

- Equation d'équilibre réduite

Force et moment au niveau de chaque nœud

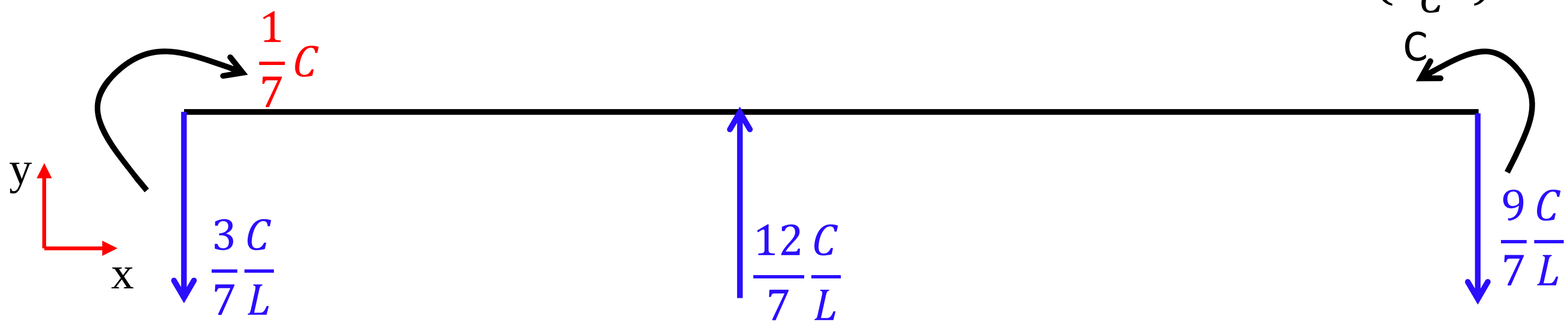
$$\frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & -0 & -12 & 6L \\ 6L & 2L^2 & 0 & \boxed{\mathbf{8L^2}} & -6L & \boxed{\mathbf{2L^2}} \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & \boxed{\mathbf{2L^2}} & -6L & \boxed{\mathbf{4L^2}} \end{bmatrix} \begin{Bmatrix} v_1 = 0 \\ \theta_1 = 0 \\ v_2 = 0 \\ \boldsymbol{\theta_2} \\ v_3 = 0 \\ \boldsymbol{\theta_3} \end{Bmatrix} = \begin{Bmatrix} R1 \\ M1 \\ R2 \\ \mathbf{0} \\ R3 \\ \boldsymbol{C} \end{Bmatrix}$$

➢ $$\frac{EI_z}{L^3}\begin{bmatrix} 8L^2 & 2L^2 \\ 2L^2 & 4L^2 \end{bmatrix}\begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ C \end{Bmatrix}$$

❖ $$\begin{Bmatrix} \theta_2 \\ \theta_3 \end{Bmatrix} = \frac{CL}{14EI_z}\begin{Bmatrix} -1 \\ 4 \end{Bmatrix}$$

- Forces nodales

$$\frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L & 0 & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 & 0 \\ -12 & -6L & 24 & -0 & -12 & 6L \\ 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ 0 & 0 & -12 & -6L & 12 & -6L \\ 0 & 0 & 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 = 0 \\ \theta_1 = 0 \\ v_2 = 0 \\ \theta_2 = \frac{-CL}{14EI_z} \\ v_3 = 0 \\ \theta_3 = \frac{4CL}{14EI_z} \end{Bmatrix} = \begin{Bmatrix} R1 \\ M1 \\ R2 \\ 0 \\ R3 \\ C \end{Bmatrix} = \begin{Bmatrix} \frac{-3}{7}\frac{C}{L} \\ \frac{-1}{7}C \\ \frac{12}{7}\frac{C}{L} \\ 0 \\ \frac{-9}{7}\frac{C}{L} \\ C \end{Bmatrix}$$



$$\checkmark \sum M/3 = \frac{-1}{7}C + 0 + C + \frac{3}{7}\frac{C}{L}\,2L - \frac{12}{7}\frac{C}{L}\,L = 0$$

$$\checkmark \sum F = -\frac{3}{7}\frac{C}{L} + \frac{12}{7}\frac{C}{L} - \frac{9}{7}\frac{C}{L} = 0$$

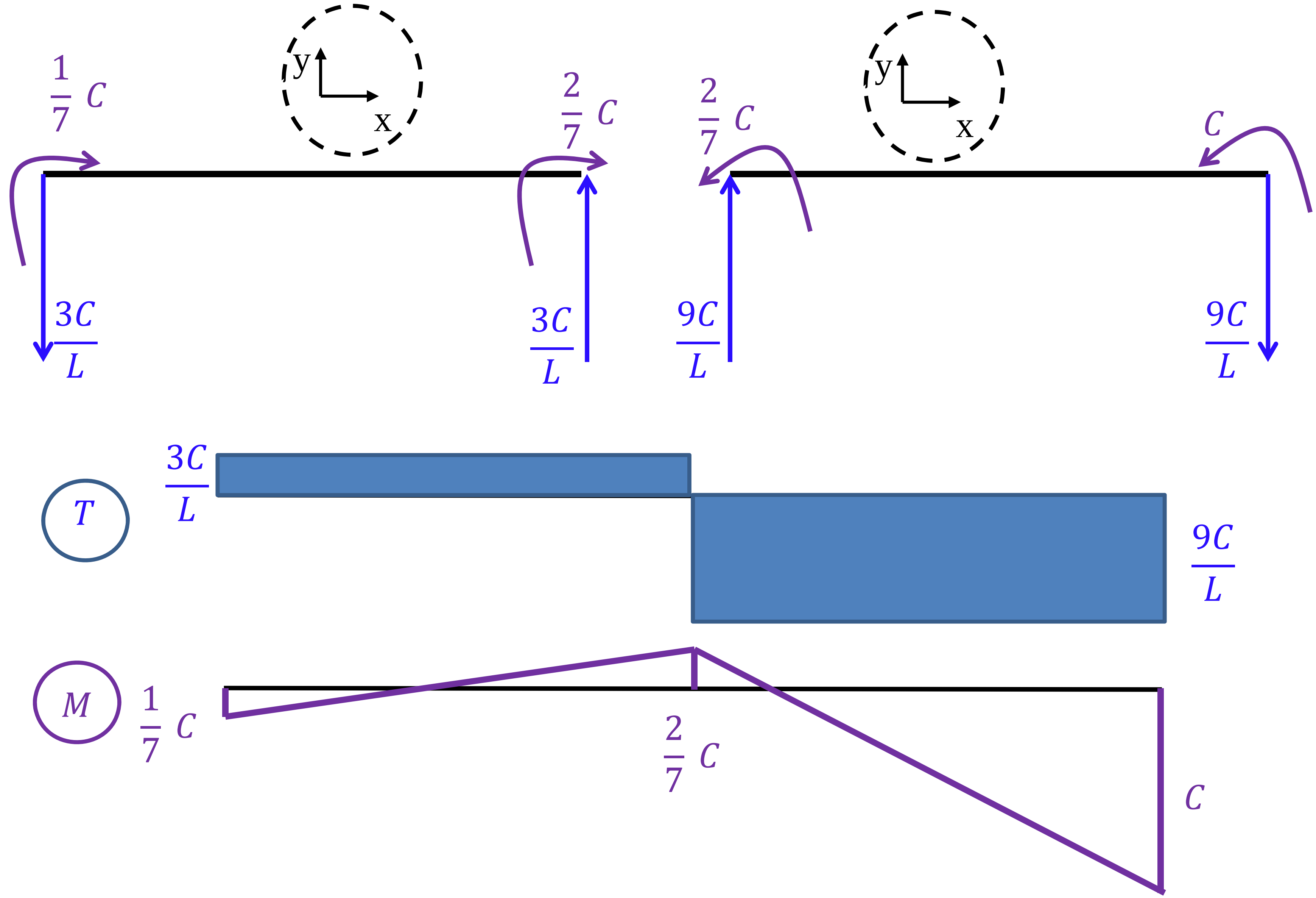
- Efforts élémentaires
  - Elément I: 1-2

✓ $$\frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_1 = 0 \\ \theta_1 = 0 \\ v_2 = 0 \\ \theta_2 = \frac{-CL}{14EI_z} \end{Bmatrix} = \begin{Bmatrix} R1 \\ M1 \\ R2 \\ M2 \end{Bmatrix} = \frac{1}{7}\begin{Bmatrix} \frac{-3C}{L} \\ -C \\ \frac{3C}{L} \\ -2C \end{Bmatrix}$$

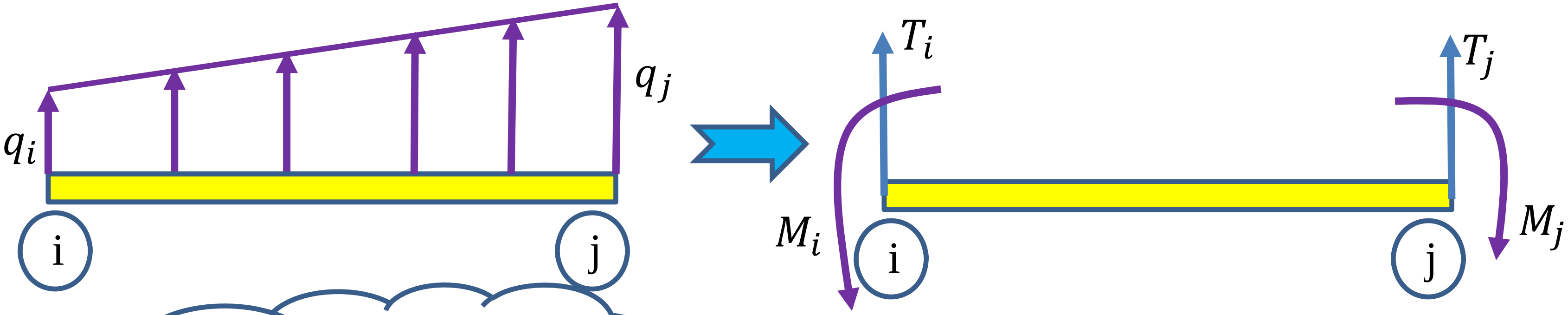
  - Elément II: 2-3

✓ $$\frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}\begin{Bmatrix} v_2 = 0 \\ \theta_2 = \frac{-CL}{14EI_z} \\ v_3 = 0 \\ \theta_3 = \frac{4CL}{14EI_z} \end{Bmatrix} = \begin{Bmatrix} R2 \\ M2 \\ R3 \\ M3 \end{Bmatrix} = \frac{1}{7}\begin{Bmatrix} \frac{9C}{L} \\ 2C \\ \frac{-9C}{L} \\ 7C \end{Bmatrix}$$

Elément finis poutre



➢ Cas d'une charge répartie



Voir le cours (l'apprendre par cœur)

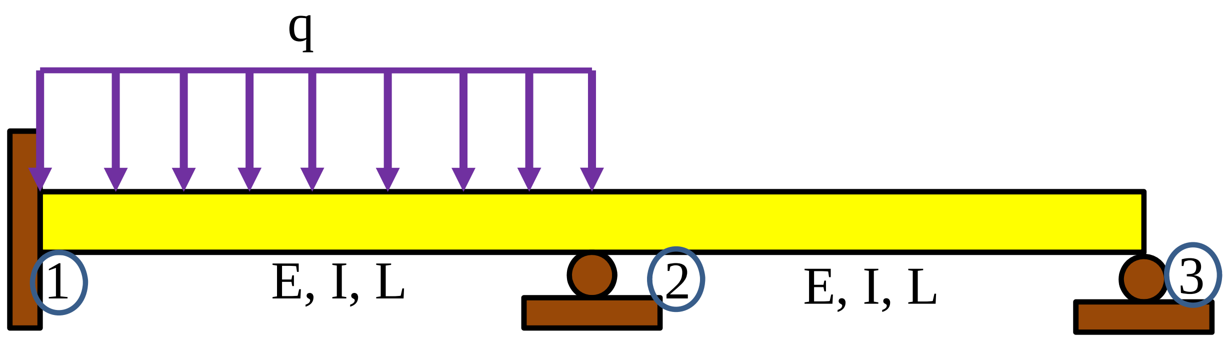
✓ $$\{f_e\} = \begin{Bmatrix} T_i \\ M_i \\ T_j \\ M_j \end{Bmatrix} = \frac{L}{60}\begin{Bmatrix} 9q_j + 21q_i \\ (2q_j + 3q_i)L \\ 21q_j + 9q_i \\ -(3q_j + 2q_i)L \end{Bmatrix}$$

Nœud i	$F_{eqi} = \int_0^L q(x)[N_1(x)]dx$	$M_{eqi} = \int_0^L q(x)[N_2(x)]dx$
Nœud j	$F_{eqj} = \int_0^L q(x)[N_3(x)]dx$	$M_{eqj} = \int_0^L q(x)[N_4(x)]dx$

Les fonctions d'interpolation s'écrivent :

$$\{N(x)\} = [1\ x\ x^2\ x^3]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{3}{L^2} & -\frac{2}{L} & \frac{3}{L^2} & -\frac{1}{L} \\ \frac{2}{L^3} & \frac{1}{L^2} & -\frac{2}{L^3} & \frac{1}{L^2} \end{bmatrix}$$

- Exemple



- Données

Elem	Ni	Nj	Ei	Ii	Li
I	1	2	E	I	L
II	2	3	E	I	L

✓ La matrice de rigidité et les conditions aux limites sont identiques à ceux du premier exemple

- Vecteur de force équivalent

$$\{f_e\} = \begin{Bmatrix} T_i \\ M_i \\ T_j \\ M_j \end{Bmatrix} = \frac{L}{60} \begin{Bmatrix} 9q_j + 21q_i \\ (2q_j + 3q_i)L \\ 21q_j + 9q_i \\ -(3q_j + 2q_i)L \end{Bmatrix}$$

Dans notre cas, pour l »élément I: $q_i = q_j = q$

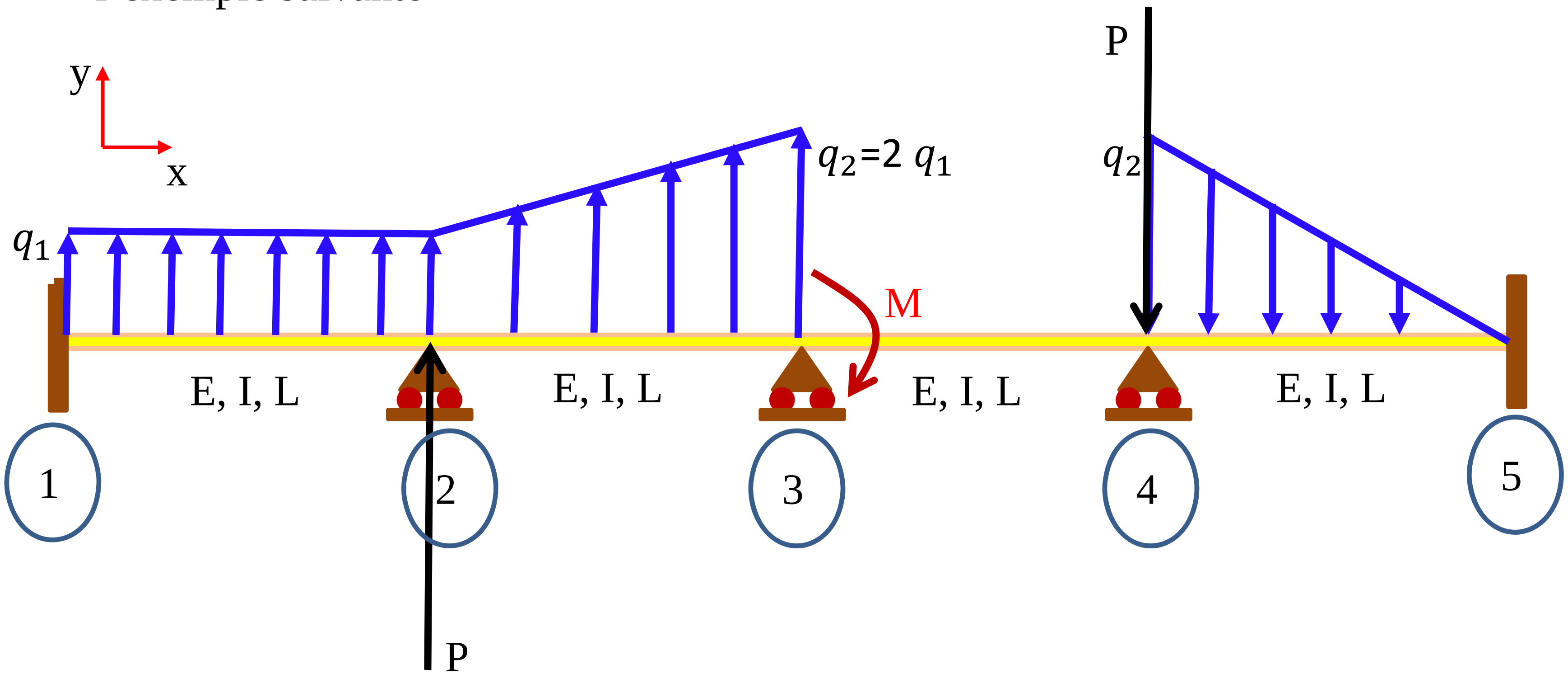
$$\{f_{eI}\} = \begin{Bmatrix} T_1 \\ M_1 \\ T_2 \\ M_2 \end{Bmatrix} = \frac{L}{60} \begin{Bmatrix} 30q \\ 5qL \\ 30q \\ -5qL \end{Bmatrix}$$

Le vecteur de force global est égale à la somme du vecteur équivalent et le vecteur de force nodale

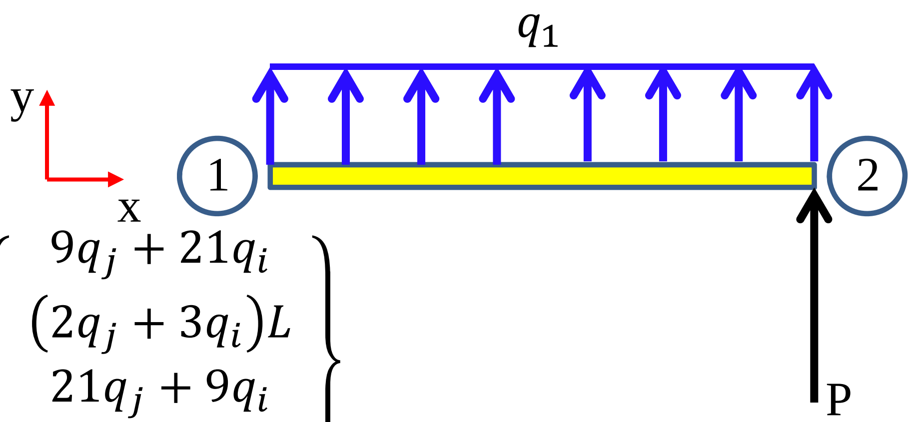
- ✓ $\{F\} = \{F_n\} + \{F_e\}$

- ❖ $\{F\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \frac{L}{60} \begin{Bmatrix} -30q \\ -5qL \\ -30q \\ 5qL \\ 0 \\ 0 \end{Bmatrix} = \frac{L}{12} \begin{Bmatrix} -6q \\ -qL \\ -6q \\ qL \\ 0 \\ 0 \end{Bmatrix}$

- Déterminer le vecteur de force globale dans l'exemple suivante



Elément I : 1-2

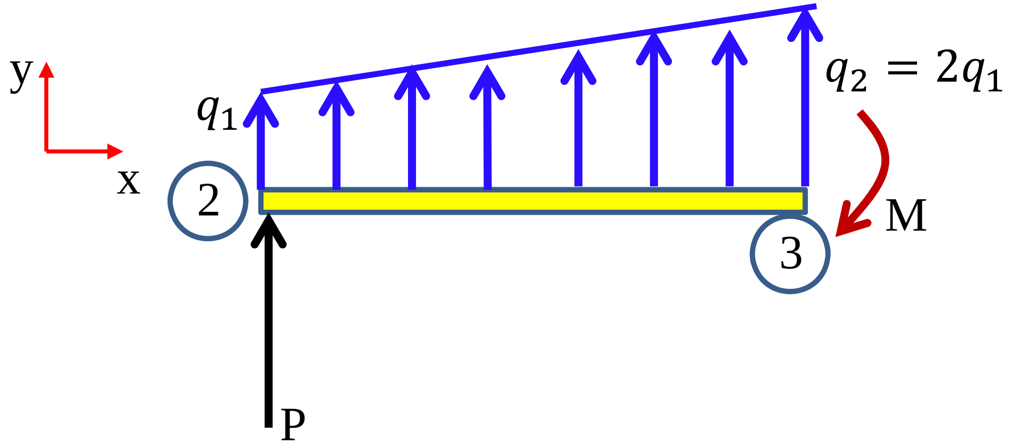


$$\{f_e\} = \begin{Bmatrix} T_i \\ M_i \\ T_j \\ M_j \end{Bmatrix} = \frac{L}{60}\begin{Bmatrix} 9q_j + 21q_i \\ (2q_j + 3q_i)L \\ 21q_j + 9q_i \\ -(3q_j + 2q_i)L \end{Bmatrix}$$

- $q_i = q_j = q_1$

❖ $\{f_{eI}\} = \begin{Bmatrix} T_{e1} \\ M_{e1} \\ T_{e2} \\ M_{e2} \end{Bmatrix} = \frac{L}{60}\begin{Bmatrix} 30q_1 \\ 5q_1 L \\ 30q_1 \\ -5q_1 L \end{Bmatrix}$ ❖ $\{f_{nI}\} = \begin{Bmatrix} T_{n1} \\ M_{n1} \\ T_{n2} \\ M_{n2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ P \\ 0 \end{Bmatrix}$

Elément II : 2-3

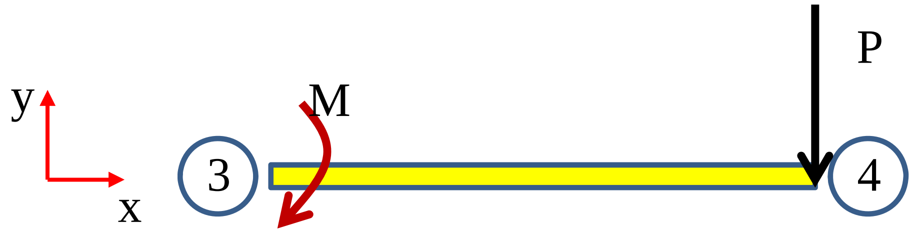


- $q_j = 2q_i = 2q_1$

- $\{f_{eII}\} = \begin{Bmatrix} T_{e2} \\ M_{e2} \\ T_{e3} \\ M_{e3} \end{Bmatrix} = \frac{L}{60} \begin{Bmatrix} 39q_1 \\ 7q_1 L \\ 51q_1 \\ -8q_1 L \end{Bmatrix}$

- $\{f_{nII}\} = \begin{Bmatrix} T_{n2} \\ M_{n2} \\ T_{n3} \\ M_{n3} \end{Bmatrix} = \begin{Bmatrix} P \\ 0 \\ 0 \\ -M \end{Bmatrix}$

Elément III : 3-4

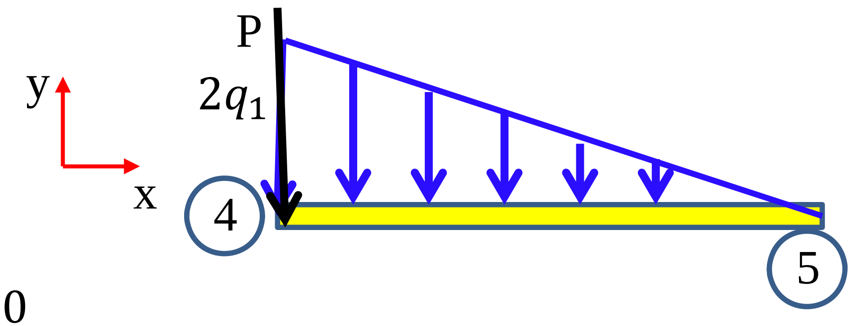


- $q_i = q_j = 0$

❖ $\{f_{eI}\} = \begin{Bmatrix} T_{e3} \\ M_{e3} \\ T_{e4} \\ M_{e4} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$

❖ $\{f_{nI}\} = \begin{Bmatrix} T_{n3} \\ M_{n3} \\ T_{n4} \\ M_{n4} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -M \\ -P \\ 0 \end{Bmatrix}$

Elément IV : 4-5



- $q_i = 2q_1$, $q_i = 0$

❖ $\{f_{eIV}\} = \begin{Bmatrix} T_{e4} \\ M_{e4} \\ T_{e5} \\ M_{e5} \end{Bmatrix} = \frac{L}{60} \begin{Bmatrix} -42q_1 \\ -6q_1 L \\ -18q_1 \\ 4q_1 L \end{Bmatrix}$

❖ $\{f_{nIV}\} = \begin{Bmatrix} T_{n4} \\ M_{n4} \\ T_{n5} \\ M_{n5} \end{Bmatrix} = \begin{Bmatrix} -P \\ 0 \\ 0 \\ 0 \end{Bmatrix}$

$$\{F\} = \{F_e\} + \{F_n\} = \begin{Bmatrix} \dfrac{q_1 L}{2} \\ \dfrac{q_1 L^2}{12} \\ \dfrac{q_1 L}{2} + \dfrac{39 q_1 L}{60} \\ -\dfrac{q_1 L^2}{12} + \dfrac{7 q_1 L^2}{60} \\ \dfrac{51 q_1 L}{60} \\ -\dfrac{8 q_1 L^2}{60} \\ -\dfrac{42 q_1 L}{60} \\ -\dfrac{6 q_1 L^2}{60} \\ -\dfrac{18 q_1 L}{60} \\ \dfrac{4 q_1 L^2}{60} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ P \\ 0 \\ 0 \\ -M \\ -P \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{q_1 L}{2} \\ \dfrac{q_1 L^2}{12} \\ \dfrac{q_1 L}{2} + \dfrac{39 q_1 L}{60} + P \\ -\dfrac{q_1 L^2}{12} + \dfrac{7 q_1 L^2}{60} \\ \dfrac{51 q_1 L}{60} \\ -\dfrac{8 q_1 L^2}{60} - M \\ -\dfrac{42 q_1 L}{60} - P \\ -\dfrac{6 q_1 L^2}{60} \\ -\dfrac{18 q_1 L}{60} \\ \dfrac{4 q_1 L^2}{60} \end{Bmatrix}$$

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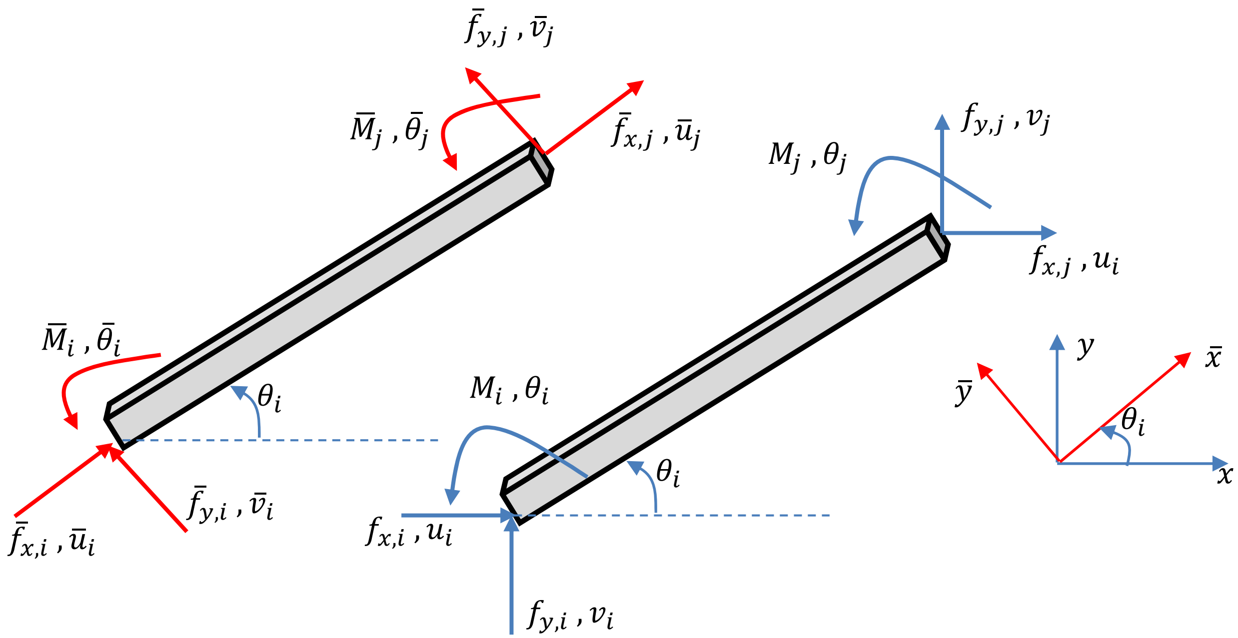
Elément finis portique



La matrice de rigidité est donnée par:

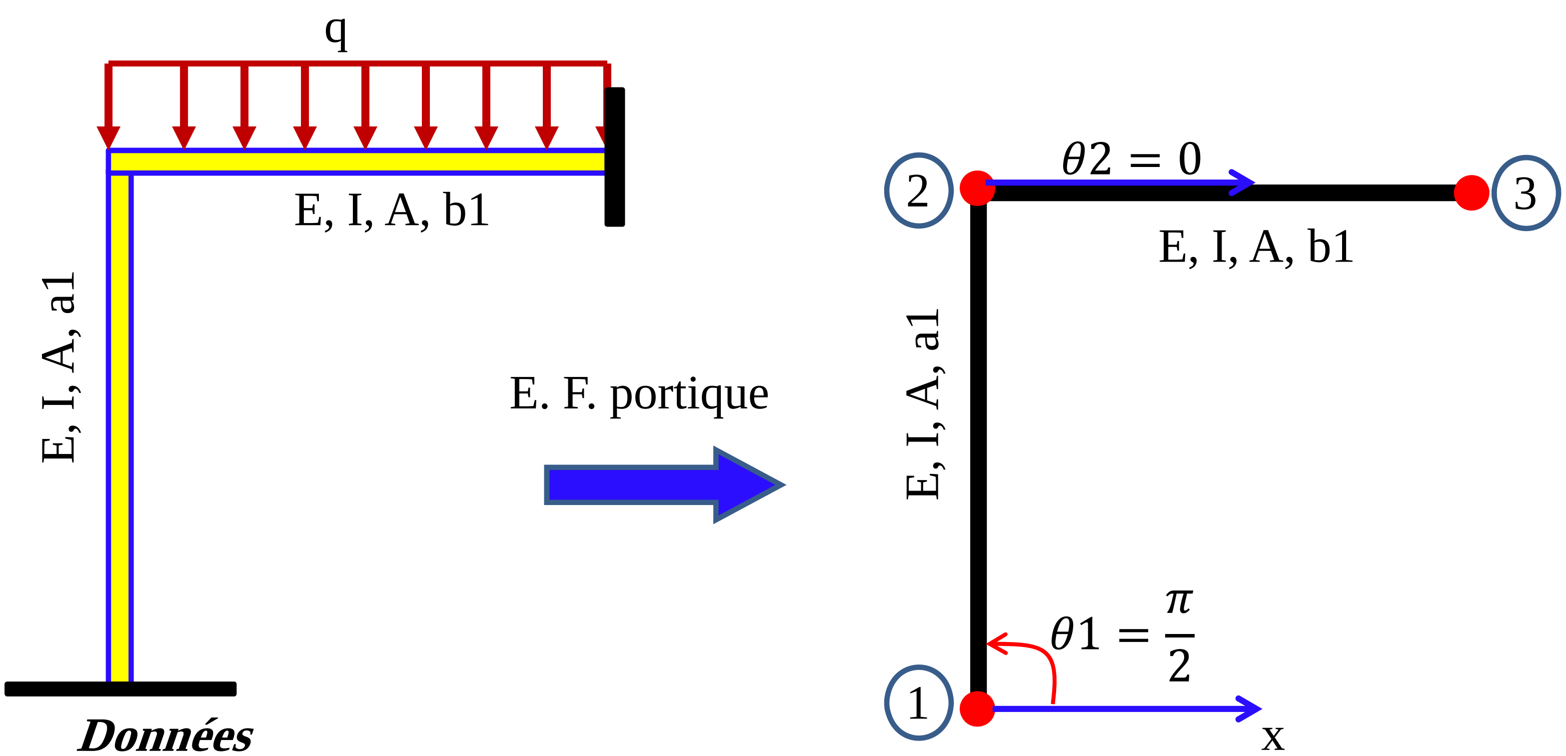
$$[k_e] = [T]^T \begin{bmatrix} a & 0 & 0 & -a & 0 & 0 \\ 0 & 12b & 6bL & 0 & -12b & 6bL \\ 0 & 6bL & 4bL^2 & 0 & -6bL & 2bL^2 \\ -a & 0 & 0 & a & 0 & 0 \\ 0 & -12b & -6bL & 0 & 12b & -6bL \\ 0 & 6bL & 2bL^2 & 0 & -6bL & 4bL^2 \end{bmatrix} [T]$$

- Dimension de $[k_e]$ = (3, 3))

✓ $$[T] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

✓ $a = \frac{EA}{L}$

✓ $b = \frac{EI}{L^3}$



Données

Elém	Ni	Nj	Ei	Li	Ai	θi	$C = \cos\theta i$	$S = \sin\theta i$
I	1	2	E	a1	A	$\frac{\pi}{2}$	0	1
II	2	3	E	b1	A	0	1	0

E=$10^6 N/m^2$, q=$\frac{10^3 N}{m^2}$, $A = 0.1\ m^2$, I $= 0.01\ m^4$ a1=3m, b1=2m

➢ Calcul des matrices des rigidité élémentaires

Elément I : 1-2

$$[k_{eI}] = [T]^T \begin{bmatrix} a & 0 & 0 & -a & 0 & 0 \\ 0 & 12b & 6bL & 0 & -12b & 6bL \\ 0 & 6bL & 4bL^2 & 0 & -6bL & 2bL^2 \\ -a & 0 & 0 & a & 0 & 0 \\ 0 & -12b & -6bL & 0 & 12b & -6bL \\ 0 & 6bL & 2bL^2 & 0 & -6bL & 4bL^2 \end{bmatrix} [T]$$

✓ $[T] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

✓ $a = \frac{EA}{a1}$

✓ $b = \frac{EI}{a1^3}$

$$[k_{eI}] = \begin{array}{c} \begin{array}{cc} \overbrace{\hspace{9em}}^{1} & \overbrace{\hspace{9em}}^{2} \end{array} \\ \begin{bmatrix} 4445 & 0 & -6667 & -4445 & 0 & 6667 \\ 0 & 3334 & 0 & 0 & -3334 & 0 \\ -6667 & 0 & 13333.5 & 6667 & 0 & 6667 \\ -4445 & 0 & 6667 & 4445 & 0 & 6667 \\ 0 & -3334 & 0 & 0 & 33334 & 0 \\ 6667 & 0 & 6667 & 6667 & 0 & 13333.5 \end{bmatrix} \end{array} \begin{array}{l} \left.\vphantom{\begin{matrix}a\\b\\c\end{matrix}}\right\} 1 \\ \left.\vphantom{\begin{matrix}a\\b\\c\end{matrix}}\right\} 2 \end{array}$$

- Calcul des matrices des rigidité élémentaires

Elément II : 2-3

$$[k_{eII}] = [T]^T \begin{bmatrix} a & 0 & 0 & -a & 0 & 0 \\ 0 & 12b & 6bL & 0 & -12b & 6bL \\ 0 & 6bL & 4bL^2 & 0 & -6bL & 2bL^2 \\ -a & 0 & 0 & a & 0 & 0 \\ 0 & -12b & -6bL & 0 & 12b & -6bL \\ 0 & 6bL & 2bL^2 & 0 & -6bL & 4bL^2 \end{bmatrix} [T]$$

- ✓ $[T] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

- ✓ $a = \frac{EA}{b1}$
- ✓ $b = \frac{EI}{b1^3}$

$$\begin{array}{cc} \overbrace{\qquad\qquad\qquad\qquad\qquad}^{2} & \overbrace{\qquad\qquad\qquad\qquad\qquad}^{3} \end{array}$$

$$[k_{eII}] = \begin{bmatrix} 50000 & 0 & 0 & -50000 & 0 & 0 \\ 0 & 15000 & 15000 & 0 & -15000 & 150000 \\ 0 & 15000 & 20000 & 0 & -15000 & 10000 \\ -50000 & 0 & 0 & 50000 & 0 & 0 \\ 0 & -15000 & -15000 & 0 & 15000 & -15000 \\ 0 & 15000 & 10000 & 0 & -15000 & 20000 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} 2 \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} 3 \end{matrix}$$

- Matrices des rigidité globale

  - Dimension de $[K]$ = ((3 x N_noeud) , (3 x N_noeud)) = (9 , 9)

$$[k_{eI}] = \begin{bmatrix} 4445 & 0 & -6667 & -4445 & 0 & 6667 \\ 0 & 3334 & 0 & 0 & -3334 & 0 \\ -6667 & 0 & 13333.5 & 6667 & 0 & 6667 \\ -4445 & 0 & 6667 & 4445 & 0 & 6667 \\ 0 & -3334 & 0 & 0 & 33334 & 0 \\ 6667 & 0 & 6667 & 6667 & 0 & 13333.5 \end{bmatrix}$$

$$[K] = \begin{array}{cc} \begin{array}{cc} \overbrace{\hphantom{50000 \quad 15000 \quad 20000}}^{2} & \overbrace{\hphantom{-50000 \quad -15000 \quad 150000}}^{3} \end{array} & \\ \begin{bmatrix} 50000 & 0 & 0 & -50000 & 0 & 0 \\ 0 & 15000 & 15000 & 0 & -15000 & 150000 \\ 0 & 15000 & 20000 & 0 & -15000 & 10000 \\ -50000 & 0 & 0 & 50000 & 0 & 0 \\ 0 & -15000 & -15000 & 0 & 15000 & -15000 \\ 0 & 15000 & 10000 & 0 & -15000 & 20000 \end{bmatrix} & \begin{array}{c} \left.\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right\} 2 \\ \left.\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right\} 3 \end{array} \end{array}$$

$$[K] = \begin{array}{cc} \begin{array}{ccc} \hphantom{00000000000}1\hphantom{00000000000} & \hphantom{00000000000}2\hphantom{00000000000} & \hphantom{00000000000}3\hphantom{00000000000} \end{array} & \\ \left[\begin{array}{ccccccccc} 4445 & 0 & -6667 & -4445 & 0 & -6667 & 0 & 0 & 0 \\ 0 & 3334 & 0 & 0 & -3334 & 0 & 0 & 0 & 0 \\ -6667 & 0 & 13333.5 & 6667 & 0 & 6667 & 0 & 0 & 0 \\ -4445 & 0 & 6667 & 54445 & 0 & 6667 & -50000 & 0 & 0 \\ 0 & -3334 & 0 & 0 & 48334 & 15000 & 0 & -15000 & 15000 \\ -6667 & 0 & 6667 & 6667 & 15000 & 33334 & 0 & -15000 & 10000 \\ 0 & 0 & 0 & -50000 & 0 & 0 & 50000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -15000 & -15000 & 0 & 15000 & -15000 \\ 0 & 0 & 0 & 0 & 15000 & 10000 & 0 & -15000 & 20000 \end{array}\right] & \begin{array}{c} \\ 1 \\ \\ \\ 2 \\ \\ \\ 3 \\ \\ \end{array} \end{array}$$

Equation du mouvement

C.A.L: $u_1=v_1=\theta_1=0$, $u_3=v_3=\theta_3=0$

$$\begin{bmatrix} 4445 & 0 & -6667 & -4445 & 0 & -6667 & 0 & 0 & 0 \\ 0 & 3334 & 0 & 0 & -3334 & 0 & 0 & 0 & 0 \\ -6667 & 0 & 13333.5 & 6667 & 0 & 6667 & 0 & 0 & 0 \\ -4445 & 0 & 6667 & \mathbf{54445} & \mathbf{0} & \mathbf{6667} & -50000 & 0 & 0 \\ 0 & -3334 & 0 & \mathbf{0} & \mathbf{48334} & \mathbf{15000} & 0 & -15000 & 15000 \\ -6667 & 0 & 6667 & \mathbf{6667} & \mathbf{15000} & \mathbf{33334} & 0 & -15000 & 10000 \\ 0 & 0 & 0 & -50000 & 0 & 0 & 50000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -15000 & -15000 & 0 & 15000 & -15000 \\ 0 & 0 & 0 & 0 & 15000 & 10000 & 0 & -15000 & 20000 \end{bmatrix} \begin{Bmatrix} u_1=0 \\ v_1=0 \\ \theta_1=0 \\ \boldsymbol{u_2} \\ \boldsymbol{v_2} \\ \boldsymbol{\theta_2} \\ u_3=0 \\ v_3=0 \\ \theta_3=0 \end{Bmatrix} = \begin{Bmatrix} Rx1 \\ Ry1 \\ M1 \\ \mathbf{0} \\ \mathbf{-1000} \\ \mathbf{-333.4} \\ 0 \\ -1000 \\ 666.7 \end{Bmatrix}$$

Equation du mouvement

C.A.L: u_1=v_1=θ_1 =0, u_3=v_3=θ_3 =0

$$\begin{bmatrix} 4445 & 0 & -6667 & -4445 & 0 & -6667 & 0 & 0 & 0 \\ 0 & 3334 & 0 & 0 & -3334 & 0 & 0 & 0 & 0 \\ -6667 & 0 & 13333.5 & 6667 & 0 & 6667 & 0 & 0 & 0 \\ -4445 & 0 & 6667 & \mathbf{54445} & \mathbf{0} & \mathbf{6667} & -50000 & 0 & 0 \\ 0 & -3334 & 0 & \mathbf{0} & \mathbf{48334} & \mathbf{15000} & 0 & -15000 & 15000 \\ -6667 & 0 & 6667 & \mathbf{6667} & \mathbf{15000} & \mathbf{33334} & 0 & -15000 & 10000 \\ 0 & 0 & 0 & -50000 & 0 & 0 & 50000 & 0 & 0 \\ 0 & 0 & 0 & 0 & -15000 & -15000 & 0 & 15000 & -15000 \\ 0 & 0 & 0 & 0 & 15000 & 10000 & 0 & -15000 & 20000 \end{bmatrix} \begin{Bmatrix} u_1 = 0 \\ v_1 = 0 \\ \theta_1 = 0 \\ \boldsymbol{u_2} \\ \boldsymbol{v_2} \\ \boldsymbol{\theta_2} \\ u_3 = 0 \\ v_3 = 0 \\ \theta_3 = 0 \end{Bmatrix} = \begin{Bmatrix} Rx1 \\ Ry1 \\ M1 \\ \mathbf{0} \\ \mathbf{-1000} \\ \mathbf{-333.4} \\ 0 \\ -1000 \\ 666.7 \end{Bmatrix}$$

Equation du mouvement réduite

- $$\begin{bmatrix} 54445 & 0 & 6667 \\ 0 & 48334 & 15000 \\ 6667 & 15000 & 33334 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1000 \\ -333.4 \end{Bmatrix}$$

$$\begin{Bmatrix} u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0.001\, m \\ -0.02\, m \\ -0.0008\, rd \end{Bmatrix}$$

- Merci pour votre attention