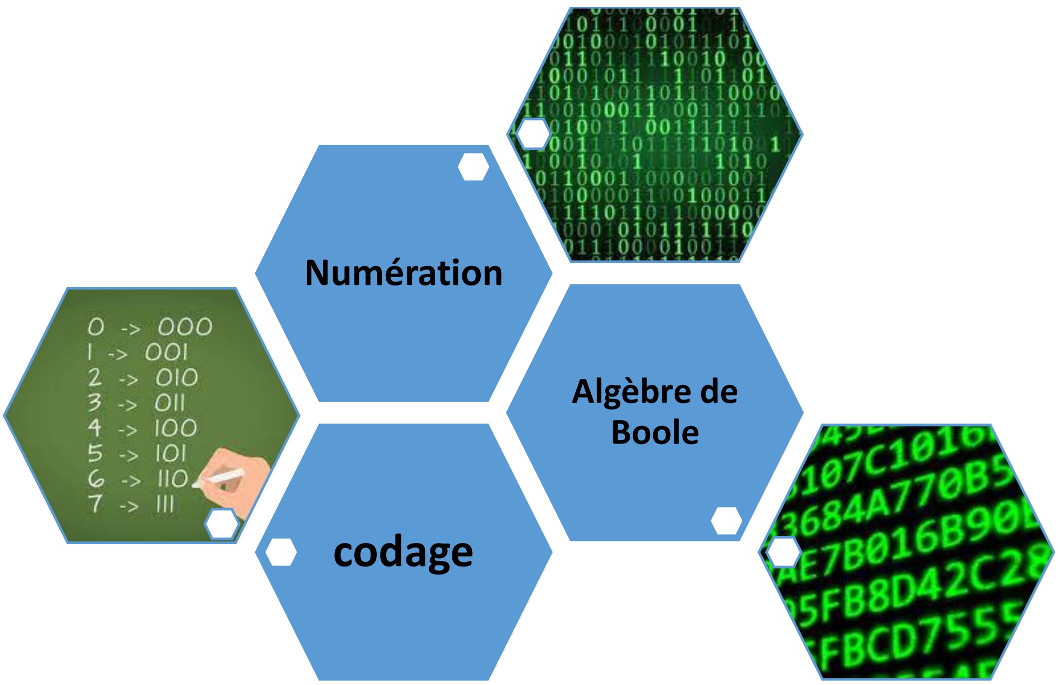
REPUBLIQUE ALGERIENNE DEMOCRATIQUE ET POPULAIRE
Ministère de l'Enseignement Supérieure et de la Recherche Scientifique
Université Djilali Bounaâma de Khemis Miliana
Faculté des sciences et techniques
Département de maths et informatique
Niveau : Licence 1ère année MI

Solution de la série de TD N°4

(Algèbre de Boole)



Exercice N°1:

Démonstration algébrique des relations suivantes :

1/ $XY + \overline{X}Z = (\overline{X} + Y)(X + Z)$

$$
\begin{aligned}
(\overline{X} + Y)(X + Z) &= \overline{X}X + \overline{X}Z + XY + YZ \\
&= XY + \overline{X}Z + (X + \overline{X})YZ \\
&= XY + \overline{X}Z + XYZ + \overline{X}YZ \\
&= XY(1 + Z) + \overline{X}Z(1 + Y) \\
&= XY + \overline{X}Z
\end{aligned}
$$

⇒La relation est vraie

2/ $XY + \overline{X}Z + YZ = XY + \overline{X}Z$

$$
\begin{aligned}
XY + \overline{X}Z + YZ &= XY + \overline{X}Z + (X + \overline{X})YZ \\
&= XY + \overline{X}Z + XYZ + \overline{X}YZ \\
&= XY(1 + Z) + \overline{X}Z(1 + Y) \\
&= XY + \overline{X}Z
\end{aligned}
$$

⇒La relation est vraie

3/ $(X + Y)(\overline{X} + Z)(Y + Z) = (X + Y)(\overline{X} + Z)$

$$
\begin{aligned}
(X + Y)(\overline{X} + Z)(Y + Z) &= (X\overline{X} + XZ + \overline{X}Y + YZ)(Y + Z) \\
&= XYZ + \overline{X}YY + YYZ + XZZ + \overline{X}YZ + YZZ \\
&= (X + \overline{X})YZ + \overline{X}Y + YZ + XZ \\
&= \overline{X}Y + YZ + XZ + X\overline{X} \\
&= \overline{X}(X + Y) + Z(X + Y) \\
&= (X + Y)(\overline{X} + Z)
\end{aligned}
$$

⇒La relation est vraie

4/ $XY + X\overline{Y}Z = XY + XZ$

$$
\begin{aligned}
XY + X\overline{Y}Z &= X(Y + \overline{Y}Z) \\
&= X((Y + \overline{Y})(Y + Z)) \\
&= X(Y + Z) \\
&= XY + XZ
\end{aligned}
$$

⇒La relation est vraie

Exercice N°2:

Représentation des fonctions sous la première et la deuxième forme canonique :

1/
$$
\begin{aligned}
X &= \bar{a}b\bar{c} + abd + bcd \\
&= \bar{a}b\bar{c}(d+\bar{d}) + a(c+\bar{c})bd + (a+\bar{a})bcd \\
&= \bar{a}b\bar{c}d + \bar{a}b\bar{c}\bar{d} + abcd + ab\bar{c}d + abcd + \bar{a}bcd \\
&= \bar{a}b\bar{c}d + \bar{a}b\bar{c}\bar{d} + ab\bar{c}d + abcd + \bar{a}bcd \\
&= \bar{a}b\bar{c}\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + ab\bar{c}d + abcd \\
&= 0000 + 0101 + 0111 + 1101 + 1111 \\
&= \sum(0,5,7,D,F) \\
&= \prod(1,2,3,4,6,8,9,A,B,C,E)
\end{aligned}
$$

➲ la première forme canonique

$$
\begin{aligned}
X_{FND}(a,b,c,d) &= \sum(0,5,7,D,F) \\
&= \bar{a}\bar{b}\bar{c}\bar{d} + \bar{a}b\bar{c}d + \bar{a}bcd + ab\bar{c}d + abcd
\end{aligned}
$$

➲ la deuxième forme canonique

$$
\begin{aligned}
X_{FNC}(a,b,c,d) &= \prod(1,2,3,4,6,8,9,A,B,C,E) \\
&= (a+b+c+\bar{d})(a+b+\bar{c}+d)(a+b+\bar{c}+\bar{d})(a+\bar{b}+c+d)(a+\bar{b}+\bar{c}+d)(\bar{a}+b+c+d) \\
&\quad (\bar{a}+b+c+\bar{d})(\bar{a}+b+\bar{c}+d)(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+\bar{b}+c+d)(\bar{a}+\bar{b}+\bar{c}+d)
\end{aligned}
$$

2/
$$
\begin{aligned}
Y &= a(b+c)(\bar{c}+\bar{d}) \\
&= (ab+ac)(\bar{c}+\bar{d}) \\
&= ab\bar{c} + ab\bar{d} + a\not{c}\bar{c} + ac\bar{d} \\
&= ab\bar{c}(d+\bar{d}) + ab(c+\bar{c})\bar{d} + a(b+\bar{b})c\bar{d} \\
&= ab\bar{c}d + ab\bar{c}\bar{d} + abc\bar{d} + ab\not{\bar{c}}\bar{d} + ab\not{c}\bar{d} + a\bar{b}c\bar{d} \\
&= a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + ab\bar{c}d + abc\bar{d} \\
&= 1010 + 1100 + 1101 + 1110 \\
&= \sum(A,C,D,E) \\
&= \prod(0,1,2,3,4,5,6,7,8,9,B,F)
\end{aligned}
$$

➲ la première forme canonique

$$
\begin{aligned}
Y_{FND}(a,b,c,d) &= \sum(A,C,D,E) \\
&= a\bar{b}c\bar{d} + ab\bar{c}\bar{d} + ab\bar{c}d + abc\bar{d}
\end{aligned}
$$

➲ la deuxième forme canonique

$$
\begin{aligned}
Y_{FNC}(a,b,c,d) &== \prod(0,1,2,3,4,5,6,7,8,9,B,F) \\
&= (a+b+c+d)(a+b+c+\bar{d})(a+b+\bar{c}+d)(a+b+\bar{c}+\bar{d})(a+\bar{b}+c+d)(a+\bar{b}+c+\bar{d})(a+\bar{b}+\bar{c}+d) \\
&\quad (a+\bar{b}+\bar{c}+\bar{d})(\bar{a}+b+c+d)(\bar{a}+b+c+\bar{d})(\bar{a}+b+\bar{c}+\bar{d})(\bar{a}+\bar{b}+\bar{c}+\bar{d})
\end{aligned}
$$

3/ $Z=(a+d)(\bar{a}+c+d)+\bar{a}\bar{b}$

⇒ **Première méthode (à partir de l'expression logique)**

$$\begin{aligned}
Z &= (a+d)(\bar{a}+c+d)+\bar{a}\bar{b}\\
&= \not{a}\bar{a}+ac+ad+\bar{a}d+cd+dd+\bar{a}\bar{b}\\
&= ac+ad+\bar{a}d+cd+d+\bar{a}\bar{b}\\
&= a(b+\bar{b})c(d+\bar{d})+a(b+\bar{b})(c+\bar{c})d+\bar{a}(b+\bar{b})(c+\bar{c})d+(a+\bar{a})(b+\bar{b})cd+(a+\bar{a})(b+\bar{b})(c+\bar{c})d+\bar{a}\bar{b}(c+\bar{c})(d+\bar{d})\\
&= abcd+abc\bar{d}+a\bar{b}cd+a\bar{b}c\bar{d}+ab\not{c}d+a\not{b}cd+ab\bar{c}d+a\bar{b}\bar{c}d+\bar{a}bcd+\bar{a}b\bar{c}d+\bar{a}\bar{b}cd+\bar{a}\bar{b}\bar{c}d+\not{a}bcd+\bar{a}bc\bar{d}+a\not{b}cd\\
&\quad +\bar{a}\not{\bar{b}}cd+ab\not{c}d+\bar{a}\not{b}cd+a\not{\bar{b}}cd+\bar{a}\not{\bar{b}}cd+a\not{b}\bar{c}d+\bar{a}\not{b}\bar{c}d+a\not{\bar{b}}\bar{c}d+\not{\bar{a}}\bar{b}\bar{c}d+\bar{a}\not{b}cd+\bar{a}\bar{b}c\bar{d}+\not{a}\bar{b}\bar{c}d+\bar{a}\bar{b}\bar{c}\bar{d}\\
&= abcd+abc\bar{d}+a\bar{b}cd+a\bar{b}c\bar{d}+ab\bar{c}d+a\bar{b}\bar{c}d+\bar{a}bcd+\bar{a}b\bar{c}d+\bar{a}\bar{b}cd+\bar{a}\bar{b}\bar{c}d+\bar{a}bc\bar{d}+\bar{a}\bar{b}c\bar{d}+\bar{a}\bar{b}\bar{c}\bar{d}\\
&= \bar{a}b\bar{c}\bar{d}+\bar{a}\bar{b}\bar{c}d+\bar{a}\bar{b}c\bar{d}+\bar{a}\bar{b}cd+\bar{a}b\bar{c}d+\bar{a}bc\bar{d}+\bar{a}bcd+a\bar{b}\bar{c}d+a\bar{b}c\bar{d}+a\bar{b}cd+ab\bar{c}d+abc\bar{d}+abcd\\
&= 0000+0001+0010+0011+0101+0110+0111+1001+1010+1011+1101+1110+1111\\
&= \sum(0,1,2,3,5,6,7,9,A,B,E,F)\\
&= \prod(4,8,C,D)
\end{aligned}$$

➲ La première forme canonique

$$\begin{aligned}
Z_{FND}(a,b,c,d) &= \sum(0,1,2,3,5,6,7,9,A,B,D,E,F)\\
&= \bar{a}b\bar{c}\bar{d}+\bar{a}\bar{b}\bar{c}d+\bar{a}\bar{b}c\bar{d}+\bar{a}\bar{b}cd+\bar{a}b\bar{c}d+\bar{a}bc\bar{d}+\bar{a}bcd+a\bar{b}\bar{c}d+a\bar{b}c\bar{d}+a\bar{b}cd+ab\bar{c}d+abc\bar{d}+abcd
\end{aligned}$$

➲ La deuxième forme canonique

$$\begin{aligned}
Z_{FNC}(a,b,c,d) &= \prod(4,8,C)\\
&= (a+\bar{b}+c+d)(\bar{a}+b+c+d)(\bar{a}+\bar{b}+c+d)
\end{aligned}$$

⇒ **Deuxième méthode (à partir de la table de vérité)**

a	b	c	d		$\bar{a}$	$\bar{b}$	$\bar{a}\bar{b}$	$(a+d)$	$(\bar{a}+c+d)$	$Z=(a+d)(\bar{a}+c+d)+\bar{a}\bar{b}$
0	0	0	0		1	1	1	0	1	1 FND
0	0	0	1		1	1	1	1	1	1 FND
0	0	1	0		1	1	1	0	1	1 FND
0	0	1	1		1	1	1	1	1	1 FND
0	1	0	0		1	0	0	0	1	0 FNC
0	1	0	1		1	0	0	1	1	1 FND
0	1	1	0		1	0	0	1	1	1 FND
0	1	1	1		1	0	0	1	1	1 FND
1	0	0	0		0	1	0	1	0	0 FNC
1	0	0	1		0	1	0	1	1	1 FND
1	0	1	0		0	1	0	1	1	1 FND
1	0	1	1		0	1	0	1	1	1 FND
1	1	0	0		0	0	0	1	0	0 FNC
1	1	0	1		0	0	0	1	1	1 FND
1	1	1	0		0	0	0	1	1	1 FND
1	1	1	1		0	0	0	1	1	1 FND

- la première forme canonique

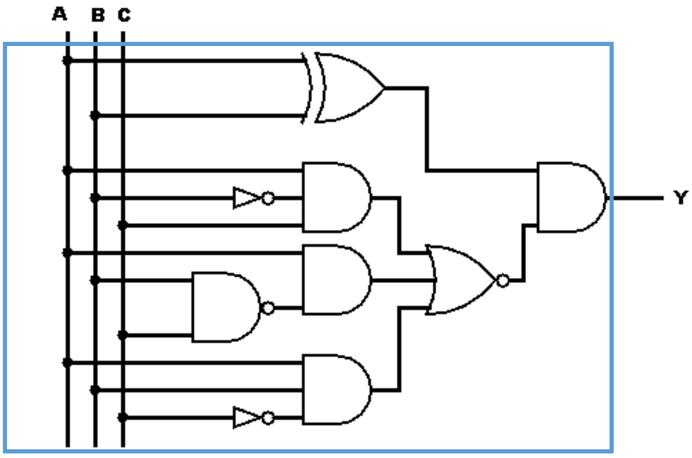
$$Z_{FND}(a,b,c,d)=\bar a b \bar c \bar d+\bar a \bar b \bar c d+\bar a \bar b c \bar d+\bar a \bar b c d+\bar a b \bar c d+\bar a b c \bar d+\bar a b c d+a \bar b \bar c d+a \bar b c \bar d+a \bar b c d+a b \bar c d+a b c \bar d+a b c d$$

- la deuxième forme canonique

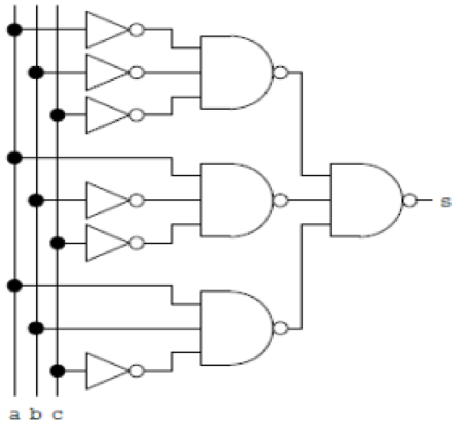
$$Z_{FNC}(a,b,c,d)=(a+\bar b+c+d)(\bar a+b+c+d)(\bar a+\bar b+c+d)$$

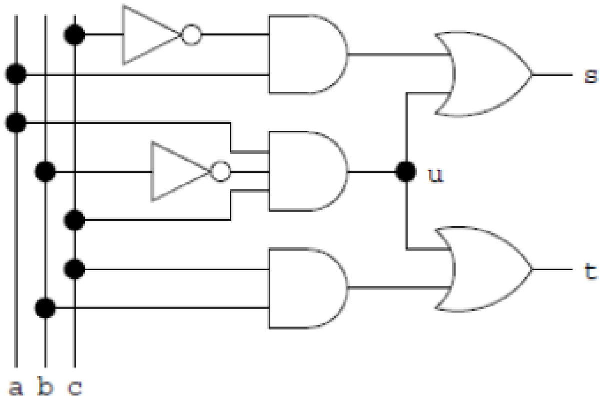
Exercice N°3:

a/ Logigramme de la fonction suivante : $Y=(A\oplus B)\left(\overline{A\bar B C+A\overline{BC}+AB\bar C}\right)$



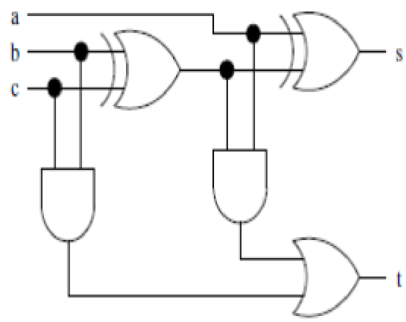
b/ Les équations de sorties :



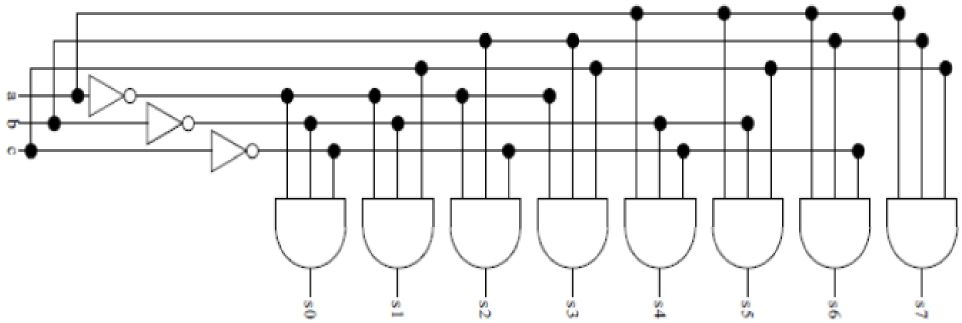


$$S(a,b,c)=\overline{\overline{\bar a \bar b \bar c}\;\overline{a \bar b \bar c}\;\overline{a b \bar c}}$$

$$\begin{cases} S(a,b,c)=a\bar c+a\bar b c \\ u(a,b,c)=a\bar b c \\ t(a,b,c)=a\bar b c+bc \end{cases}$$



$$\begin{cases} S(a,b,c)=a\oplus(b\oplus c) \\ t(a,b,c)=a(b\oplus c)+bc \end{cases}$$



$$\begin{cases} S_0(a,b,c) = \bar{a}\bar{b}\bar{c} \\ S_1(a,b,c) = \bar{a}\bar{b}c \\ S_2(a,b,c) = \bar{a}b\bar{c} \\ S_3(a,b,c) = \bar{a}bc \\ S_4(a,b,c) = a\bar{b}\bar{c} \\ S_5(a,b,c) = a\bar{b}c \\ S_6(a,b,c) = ab\bar{c} \\ S_7(a,b,c) = abc \end{cases}$$

Exercice N°4:

Les équations simplifiées en utilisant les tableaux de KARNAUGH

AB \ CD	00	01	11	10
00	0	1	1	0
01	1	1	1	1
11	0	1	1	0
10	0	1	1	0

$F(A,B,C,D) = B + \bar{C}D$

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	1

$F(A,B,C,D) = BD + \bar{B}\bar{D}$

AB \ CD	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

$F(A,B,C,D) = \bar{C}D + C\bar{D}$

AB \ CD	00	01	11	10
00	0	1	0	1
01	0	1	0	1
11	1	1	1	1
10	1	1	1	1

$F(A,B,C,D) = A\bar{B} + \bar{A}B + C$

CD \ AB	00	01	11	10
00	0	1	1	0
01	1	1	1	1
11	1	1	1	1
10	0	1	1	0

$F(A,B,C,D) = B + D$

CD \ AB	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	0	1	0
10	1	1	1	1

$F(A,B,C,D) = AB + C\overline{D}$

CD \ AB	00	01	11	10
00	1	0	1	1
01	1	0	1	1
11	0	0	1	1
10	1	1	1	1

$F(A,B,C,D) = A + \overline{B}\overline{C} + C\overline{D}$

CD \ AB	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	1

$F(A,B,C,D) = B + \overline{D}$

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$F(A,B,C,D) = \overline{A}D + \overline{C}D + \overline{B}\overline{C}$

CD \ AB	00	01	11	10
00	0	0	1	0
01	1	0	1	1
11	1	1	1	1
10	0	0	1	0

$F(A,B,C,D) = AB + CD + \overline{A}D$

Exercice N°5: Table de vérité

A	B	C	D	[redacted]	F
0	0	0	0	[redacted]	1
0	0	0	1	[redacted]	1
0	0	1	0	[redacted]	1
0	0	1	1	[redacted]	1
0	1	0	0	[redacted]	1
0	1	0	1	[redacted]	1
0	1	1	0	[redacted]	1
0	1	1	1	[redacted]	1
1	0	0	0	[redacted]	1
1	0	0	1	[redacted]	1
1	0	1	0	[redacted]	1
1	0	1	1	[redacted]	1
1	1	0	0	[redacted]	1
1	1	0	1	[redacted]	1
1	1	1	0	[redacted]	1
1	1	1	1	[redacted]	1

➦ Simplification par la méthode de karnaugh

AB / CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$F(A,B,C,D) = 1$$

➦ Simplification algébrique

$$\begin{aligned}
F(A,B,C,D) &= A\overline{C} + ABD + AB\overline{D} + \overline{A}C + \overline{AB} \\
&= A\overline{C} + AB(D+\overline{D}) + \overline{A}C + \overline{A} + \overline{B} \\
&= A\overline{C} + (AB+\overline{B}) + \overline{A}C + \overline{A} \\
&= A\overline{C} + ((A+\overline{B})(B+\overline{B})) + \overline{A}C + \overline{A} \\
&= A\overline{C} + \overline{A} + A + \overline{B} + \overline{A}C \\
&= A\overline{C} + 1 + \overline{B} + \overline{A}C \\
&= 1
\end{aligned}$$

Exercice N°6:

$$F(A,B,C,D) = ABCD + AB\overline{C}D + A\overline{B}C + \overline{A}BCD + AB + C\overline{D}$$

➦ Table de vérité

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

➦ La 1[er] et la 2[ème] formes canoniques de F (FND et FNC)

$$F_{FND}(A,B,C,D)=\overline{A}\,\overline{B}C\overline{D}+\overline{A}BC\overline{D}+\overline{A}BCD+A\overline{B}C\overline{D}+A\overline{B}CD+AB\overline{C}\,\overline{D}+AB\overline{C}D+ABC\overline{D}+ABCD$$

$$F_{FNC}(A,B,C,D)=(A+B+C+\overline{D})(A+B+\overline{C}+\overline{D})(A+\overline{B}+C+D)(A+\overline{B}+C+\overline{D})(\overline{A}+B+C+D)(\overline{A}+B+C+\overline{D})$$

➦ Simplification de F en utilisant les lois de l'algèbre de Boole

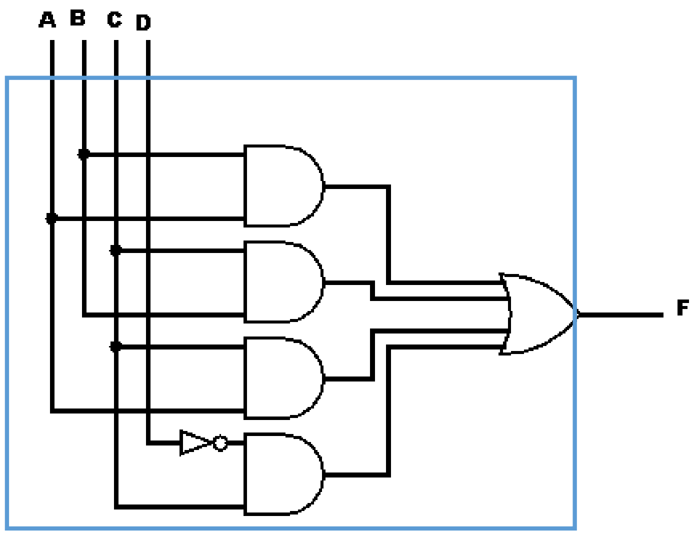
$$\begin{aligned}
F(A,B,C,D) &= ABCD+AB\overline{C}D+A\overline{B}C+\overline{A}BCD+AB+C\overline{D}\\
&= ABDC+ABCD+AB\overline{C}D+A\overline{B}C+\overline{A}BCD+AB+C\overline{D}\\
&= ABD(C+\overline{C})+A\overline{B}C+BCD(A+\overline{A})+AB+C\overline{D}\\
&= ABD+A\overline{B}C+BCD+AB+C\overline{D}\\
&= AB(1+D)+A\overline{B}C+BCD+C\overline{D}\\
&= A(B+\overline{B}C)+C(BD+\overline{D})\\
&= A((B+\overline{B})(B+C))+C((B+\overline{D})(D+\overline{D}))\\
&= A(B+C)+C(B+\overline{D})\\
&= AB+AC+BC+C\overline{D}
\end{aligned}$$

➦ Simplification de F en utilisant la table de Karnaugh

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	1	1	1
10	1	1	1	1

$$F(A,B,C,D)=AB+BC+AC+C\overline{D}$$

➦ les logigrammes simplifiés



Exercice N°7:

$$F(A,B,C) = AB\overline{C} + A\overline{B} + \overline{A}\,\overline{B}\,\overline{C} + \overline{B}C$$

- Table de vérité

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

- Les formes canoniques

$$F_{FND}(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C}$$

$$F_{FNC}(A,B,C) = \left(A+\overline{B}+C\right)\left(A+\overline{B}+\overline{C}\right)\left(\overline{A}+\overline{B}+\overline{C}\right)$$

- Simplification par la table de karnaugh

AB \ C	00	01	11	10
0	1	0	1	1
1	1	0	0	1

$$F(A,B,C) = \overline{B} + A\overline{C}$$

- Simplification par la méthode de quine MC-cluskey

$$F_{FND}(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C}$$
$$= 000 + 001 + 100 + 101 + 110$$

000✓	00X✓	X0X
001✓	X00✓	~~X0X~~
100✓	X01✓	
101✓	10X✓	
110✓	1X0	

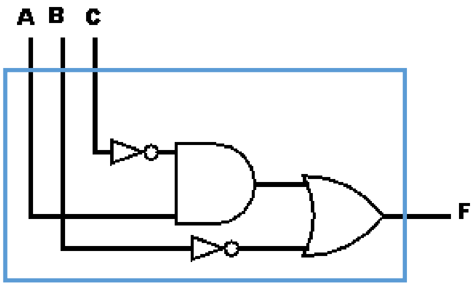
	000	001	100	101	110
1X0	X		X		⊗
X0X	X	⊗	X	⊗	

$$F_{FND}(A,B,C) = 1X0 + X0X$$
$$= A\overline{C} + \overline{B}$$

➦ Simplification algébrique :

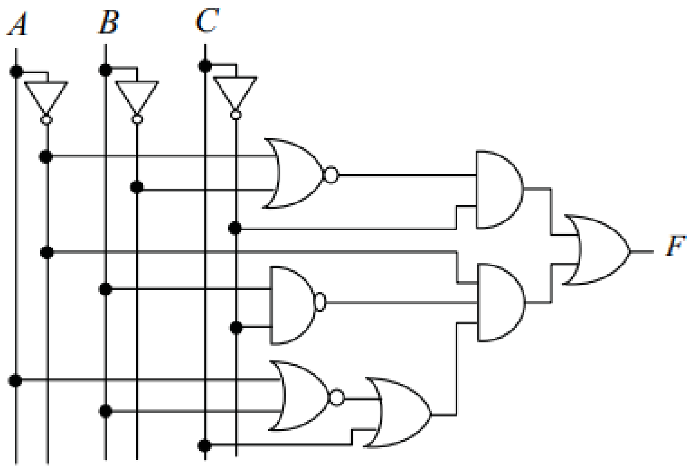
$$
\begin{aligned}
F_{FND}(A,B,C) &= AB\overline{C} + A\overline{B} + \overline{A}\,\overline{B}\,\overline{C} + \overline{B}C \\
&= A\left(B\overline{C} + \overline{B}\right) + \overline{B}\left(\overline{A}\,\overline{C} + C\right) \\
&= A\left(\left(\overline{C} + \overline{B}\right)\left(B + \overline{B}\right)\right) + \overline{B}\left(\left(\overline{A} + C\right)\left(\overline{C} + C\right)\right) \\
&= A\left(\overline{C} + \overline{B}\right) + \overline{B}\left(\overline{A} + C\right) \\
&= A\overline{C} + A\overline{B} + \overline{A}\,\overline{B} + \overline{B}C \\
&= A\overline{C} + \overline{B}\left(A + \overline{A}\right) + \overline{B}C \\
&= A\overline{C} + \overline{B} + \overline{B}C \\
&= A\overline{C} + \overline{B}(1 + C) \\
&= A\overline{C} + \overline{B}
\end{aligned}
$$

➦ Les logigrammes simplifiés



Exercice N°8:

➢ L'expression logique :



$$
\begin{aligned}
F(A,B,C) &= \overline{\left(\overline{A} + \overline{B}\right)}\,\overline{C} + \overline{A}\,\overline{B\overline{C}}\left(\left(\overline{A + B}\right) + C\right) \\
&= AB\overline{C} + \overline{A}\left(\overline{B} + C\right)\left(\overline{A}\,\overline{B} + C\right) \\
&= AB\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,\overline{B}C + \overline{A}C
\end{aligned}
$$

- Table de vérité :

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

- Les formes canoniques

$$F_{FND}(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}BC + AB\overline{C}$$
$$F_{FNC}(A,B,C) = (A+\overline{B}+C)(\overline{A}+B+C)(\overline{A}+B+\overline{C})(\overline{A}+\overline{B}+\overline{C})$$

- Simplification par la table de Karnaugh

AB / C	00	01	11	10
0	1	0	1	0
1	1	1	0	0

$$F(A,B,C) = AB\overline{C} + \overline{A}\,\overline{B} + \overline{A}C$$

- Simplification par la méthode de quine MC-Cluskey

$$F_{FND}(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}BC + AB\overline{C}$$
$$= 000 + 001 + 011 + 110$$

000 ✓	00X
001 ✓	0X1
011 ✓	
110	

	000	001	011	110
110				ⓥ
00X	ⓥ	✓		
0X1		✓	ⓥ	

$$F_{FND}(A,B,C) = 110 + 00X + 0X1$$
$$= AB\overline{C} + \overline{A}\,\overline{B} + \overline{A}C$$

Simplification algébrique :

$$F(A,B,C)=\overline{\left(\overline{A}+\overline{B}\right)}\,\overline{C}+\overline{A}\,\overline{B\overline{C}}\left(\left(\overline{A+B}\right)+C\right)$$
$$=AB\overline{C}+\overline{A}\left(\overline{B}+C\right)\left(\overline{A}\,\overline{B}+C\right)$$
$$=AB\overline{C}+\overline{A}\,\overline{B}+\overline{A}\,\overline{B}C+\overline{A}C$$
$$=AB\overline{C}+\overline{A}\,\overline{B}\left(1+C\right)+\overline{A}C$$
$$=AB\overline{C}+\overline{A}\,\overline{B}+\overline{A}C$$

Exercice N°9:

Réalisation d'un circuit complément à 1 à 4 bits.

- La table de vérité

X_3	X_2	X_1	X_0		C_3	C_2	C_1	C_0
0	0	0	0		1	1	1	1
0	0	0	1		1	1	1	0
0	0	1	0		1	1	0	1
0	0	1	1		1	1	0	0
0	1	0	0		1	0	1	1
0	1	0	1		1	0	1	0
0	1	1	0		1	0	0	1
0	1	1	1		1	0	0	0
1	0	0	0		0	1	1	1
1	0	0	1		0	1	1	0
1	0	1	0		0	1	0	1
1	0	1	1		0	1	0	0
1	1	0	0		0	0	1	1
1	1	0	1		0	0	1	0
1	1	1	0		0	0	0	1
1	1	1	1		0	0	0	0

- Les équations de sortie

X_3X_2 \ X_1X_0	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

$$C_3(X_3,X_2,X_1,X_0)=\overline{X}_3$$

X_3X_2 \ X_1X_0	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

$$C_2(X_3,X_2,X_1,X_0)=\overline{X}_2$$

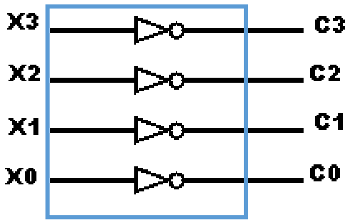
X_1X_0 \ X_3X_2	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$$C_1(X_3, X_2, X_1, X_0) = \overline{X}_1$$

X_1X_0 \ X_3X_2	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$$C_0(X_3, X_2, X_1, X_0) = \overline{X}_0$$

➦ Le logigramme simplifié



Complément à 1 à 4 bits.