Correction de la serie N°2

EXO N° 1

- $\int_1^4 \frac{1-\sqrt{x}}{\sqrt{x}}dx = \int_1^4 \left(\frac{1}{\sqrt{x}} - 1\right)dx = \int_1^4 \frac{1}{\sqrt{x}}dx - \int_1^4 dx = \left[2\sqrt{x}\right]_1^4 - \left[x\right]_1^4$

$$\int_1^4 \frac{1-\sqrt{x}}{x}dx = \left(2\sqrt{4} - 2\sqrt{1}\right) - (4-1) = 2 - 3 = -1.$$

- $\int_1^3 \frac{x + \sqrt[3]{x^2}}{x}dx = \int_1^3 \left(1 + \frac{\sqrt[3]{x^2}}{x}\right)dx = \int_1^3 dx + \int_1^3 x^{2/3} \times x^{-1} dx$

$$= \left[x\right]_1^3 + \frac{3}{2}\left[\sqrt[3]{x^2}\right]_1^3$$

- $\int (\sqrt[4]{x^3} + 1)(x+1)dx = \int x^{7/4}dx + \int x^{3/4}dx + \int x\,dx + \int dx.$

$$= \frac{x^{7/4+1}}{7/4+1} + \frac{x^{3/4+1}}{\frac{3}{4}+1} + \frac{x^2}{2} + x + C \quad / C \in \mathbb{R}$$

- $\int \frac{e^x - 1}{e^x}dx = \int \left(1 - \frac{1}{e^x}\right)dx = \int dx + \int -e^{-x}dx = x + e^{-x} + C \ / C \in \mathbb{R}$

EXO N°2

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \tan(x)\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin x}{\cos x}dx = -\int_{\frac{\sqrt{3}}{2}}^{\sqrt{2}/2} \frac{1}{u}du = -\left[\ln|u|\right]_{\frac{\sqrt{3}}{2}}^{\frac{\sqrt{2}}{2}} = \frac{1}{2}\ln\left(\frac{3}{2}\right)$$

CV : on pose $u = \cos x \Rightarrow \frac{du}{dx} = -\sin x$ donc $dx = -\frac{du}{\sin x}$

qdn $\frac{\pi}{6} \leq x \leq \frac{\pi}{4}$; $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

- $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int \frac{1}{\sqrt{x}} \cdot e^{\sqrt{x}} dx = 2\int e^{u} du = 2e^{u} + C = 2e^{\sqrt{x}} + C.$

C.V : on pose $u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$dx = 2\sqrt{x}.\, du.$

- $\int \frac{\cos x}{\sqrt{1 - \sin^2 x}} dx = \int \frac{1}{\sqrt{1-u^2}} du = \arcsin(u) + C = \arcsin(\sin x) + C$

$C \in \mathbb{R}$

on pose $u = \sin x \Rightarrow \frac{du}{dx} = \cos x$

$dx = \frac{du}{\cos x}$

- $\int_1^2 \frac{\ln^2 x}{x} dx = \int_1^2 \frac{1}{x} \ln^2 x \, dx = \int_0^{\ln 2} u^2 du = \left.\frac{u^3}{3}\right]_0^{\ln 2} = \frac{1}{3}(\ln 2)^3$

CV : $u = \ln x \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x} dx.$

- $\int \frac{6x+7}{3x^2+7x-4} dx = \int \frac{du}{u} = \ln|u| + C = \ln|3x^2+7x-4| + C$

C.V : on pose. $u = 3x^2+7x-4 \Rightarrow \frac{du}{dx} = 6x+7.$

$du = (6x+7)dx$

- $\int \frac{x}{x^2+4} dx = \frac{1}{2}\int \frac{1}{u^2+4} du = \frac{1}{2}\left[\int \frac{1}{4(\frac{u^2}{4}+1)} du\right] = \frac{1}{8}\int \frac{1}{(\frac{u}{2})^2+1} du$

$= \frac{1}{8} \arctan\left(\frac{u}{2}\right) + C$

$u = x^2 \Rightarrow \frac{du}{dx} = 2x$

$= \frac{1}{8} \arctan\left(\frac{x^2}{2}\right) + C, \; C \in \mathbb{R}$

EXO N° 3

- $\int_1^4 \ln x\,dx = \int_1^4 1 . \ln x\,dx = \left[x\ln x\right]_1^4 - \int_1^4 x \times \frac{1}{x}\,dx = \left[x\ln x\right]_1^4 - \left[x\right]_1^4$

$= 4\ln 4 - (4-1) = 4\ln 4 - 3.$

$u' = 1 \Rightarrow u = x.$

$v = \ln x \Rightarrow v' = \frac{1}{x}.$

- $\int x\ln x\,dx = \left[\frac{x^2}{2}\ln x\right] - \int \frac{x^2}{2} \times \frac{1}{x}\,dx = \left[\frac{x^2}{2}\ln x\right] - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C \quad / C \in \mathbb{R}$

$u' = x \Rightarrow u = \frac{x^2}{2}.$

$v = \ln x \Rightarrow v' = \frac{1}{x}.$

- $\int_0^1 x^2 e^x\,dx = \left[x^2 e^x\right]_0^1 - \int_0^1 2x e^x\,dx = \left[x^2 e^x\right]_0^1 - 2\int_0^1 x e^x\,dx$

$= 1e^1 - 2 = e^1 - 2.$

$u = x^2 \Rightarrow u' = 2x$

$v' = e^x \Rightarrow v = e^x$

on procède à une deuxième intégration par partie pour calculer $\int_0^1 x e^x\,dx$

$\int_0^1 x e^x\,dx = \left[x e^x\right]_0^1 - \int_0^1 e^x\,dx = x e^x\Big]_0^1 - \left[e^x\right]_0^1 = e^1 - (e^1 - e^0) = e^1 - e^1 + e^0$

$= e^0 = 1.$

$u = x \Rightarrow u' = 1$

$v' = e^x \Rightarrow v = e^x$

• $\int \arccos x \, dx = \int 1 . \arccos x \, dx = [x \arccos(x)] - \int x . \left(-\frac{1}{\sqrt{1-x^2}}\right) dx$

$= [x \arccos(x)] + \int \frac{x}{\sqrt{1-x^2}} dx =$

$u' = 1 \Rightarrow u = x$.

$v = \arccos(x) \Rightarrow v' = \frac{-1}{\sqrt{1-x^2}}$

on procède à un changement de variable pour calculer.

$\int \frac{x}{\sqrt{1-x^2}} dx$

on pose : $U = 1 - x^2 \Rightarrow \frac{du}{dx} = -2x$.

$du = -2x \, dx$. C.A.D : $x \, dx = -\frac{1}{2} du$.

$\int \frac{x}{\sqrt{1-x^2}} dx = \int -\frac{1}{2} \frac{1}{\sqrt{u}} du = -\frac{1}{2} \int u^{-1/2} du$

$= -\frac{1}{2} \left[\frac{u^{-1/2+1}}{-1/2+1} \right] + C$

$= -\frac{1}{2} \times 2 \sqrt{u} + C$

$= -\sqrt{1-x^2} + C$

$\int \arccos(x) dx = [x \arccos(x)] - \sqrt{1-x^2} + C \quad , C \in \mathbb{R}$.

• $\int x^2 \sin 3x \, dx = \left[-\frac{1}{3} x^2 \cos 3x\right] - \int -\frac{2}{3} x \cos 3x \, dx$

$= \left(-\frac{1}{3} x^2 \cos 3x\right) + \frac{2}{3} \int x \cos 3x \, dx$

$u = x^2 \Rightarrow u' = 2x$.

$v' = \sin 3x \Rightarrow v = -\frac{1}{3} \cos 3x$.

on calcule par une intégration par partie $\int x \cos 3x \, dx$

$$\int x\cos 3x\,dx = \left[\frac{1}{3}x\sin 3x\right] - \int \frac{1}{3}\sin 3x\,dx$$

$u = x \Rightarrow u' = 1$

$v' = \cos 3x \Rightarrow v = \frac{1}{3}\sin 3x$.

$$\int x\cos 3x\,dx = \left(\frac{1}{3}x\sin 3x\right) - \frac{1}{3}\int \sin 3x\,dx$$

$$= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C.$$

$$\int x^2\sin 3x\,dx = -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left(\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C\right)$$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C' \;/ C'\in\mathbb{R}$$

EXO 4 $\quad \frac{P}{Q}$

$$\frac{x^2}{x+1} = \frac{x^2+1-1}{x+1} = \frac{x^2-1}{x+1} + \frac{1}{x+1} = x-1+\frac{1}{x+1}. \quad (\deg(P) > \deg(Q))$$

$$\int_0^2 \frac{x^2}{x+1}\,dx = \int_0^2\left(x-1+\frac{1}{x+1}\right)dx = \frac{1}{2}\left[x^2\right]_0^2 - \left[x\right]_0^2 + \left.\ln|x+1|\right]_0^2$$

$$= \frac{1}{2}\times 4 - 2 + \ln 2 = \ln 2.$$

$$\frac{x^2-1}{x^2+1} = \frac{x^2+1-1-1}{x^2+1} = \frac{x^2+1}{x^2+1} - \frac{2}{x^2+1} = 1 - \frac{2}{x+1} \quad (\deg(P) = \deg(Q))$$

$$\int_0^1 \frac{x^2-1}{x^2+1}\,dx = \int_0^1\left(1-\frac{2}{x+1}\right)dx = \int_0^1 dx - 2\int_0^1\frac{1}{x+1}\,dx$$

$$= \left.x\right]_0^1 - 2\left[\ln|x+1|\right]_0^1 = 1 - 2\ln 2$$

- $\displaystyle\int \frac{dx}{x^2-3x-4}$

x^2-3x-4

$\Delta = 9+16 = 25$

$x_1 = \frac{3+5}{2} = 4$

$x_2 = \frac{3-5}{2} = -1$

$x^2-3x-4 = (x+1)(x-4)$

$$\frac{1}{x^2-3x-4} = \frac{A}{x+1} + \frac{B}{x-4} = \frac{A(x-4)+B(x+1)}{(x+1)(x-4)}$$

$$= \frac{(A+B)x - 4A + B}{(x+1)(x-4)}$$

$A+B=0 \implies A=-B$.

$-4A+B=1 \implies -4(-B)+B = 1$.! $\implies B = 1/5$. et $A = -1/5$.

$$\int \frac{dx}{x^2-3x-4} = -\frac{1}{5}\int \frac{1}{x+1}\,dx + \frac{1}{5}\int \frac{1}{x-4}\,dx$$

$$= -\frac{1}{5}\ln|x+1| + \frac{1}{5}\ln|x-4| + C.$$

- $\displaystyle\int \frac{dx}{x^2+x+1} = \frac{4}{3}\int \frac{1}{\left(\frac{2x+1}{\sqrt{3}}\right)^2+1}\,dx = \frac{4}{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right) + C$

x^2+x+1

$\Delta = (1)^2 - 4(1)(1)$

$$x^2+x+1 = \left(x+\frac{1}{2}\right)^2 - \frac{1}{4} + 1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} = \frac{3}{4}\left[\frac{(x+1/2)^2}{3/4} + 1\right]$$

$$\frac{1}{x^2+x+1} = \frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{1}{\frac{3}{4}\left[\frac{(x+1/2)^2}{\frac{3}{4}}+1\right]}$$

$$= \frac{4}{3} \times \frac{1}{\left[\left(\frac{2x+1}{\sqrt{3}}\right)^2+1\right]}$$

P

$$\int \frac{x}{(x-2)^2}\,dx$$

$$\frac{x}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2} = \frac{A(x-2)+B}{(x-2)^2} = \frac{Ax-2A+B}{(x-2)^2}$$

$A = 1$

$-2A+B = 0 \Rightarrow -2+B=0$ c.a.d $\boxed{B=2}$

$$\int \frac{x}{(x-2)^2}\,dx = \int \frac{1}{x-2}\,dx + \int \frac{2}{(x-2)^2}\,dx.$$

$$= \ln|x-2| + 2\int u^{-2}\,du$$

$u = x-2$

$\frac{du}{dx} = 1.$

$$= \ln|x-2| - \frac{2}{u} + C$$

$$= \ln|x-2| - \frac{2}{x-2} + C$$

- $$\int \frac{2x+3}{x^2+x+2}\,dx$$

$$\frac{2x+3}{x^2+x+2} = \frac{2x+1-1+3}{x^2+x+2} = \frac{2x+1}{x^2+x+2} + \frac{2}{x^2+x+2}$$

$$\int \frac{2x+3}{x^2+x+2}\,dx = \int \frac{2x+1}{x^2+x+2}\,dx + 2\int \frac{1}{x^2+x+2}\,dx.$$

$$\int \frac{2x+1}{x^2+x+2}\,dx = \ln|x^2+x+2| + C_1$$ (changement de variable $u=x^2+x+2$)

$$\int \frac{1}{x^2+x+2}\,dx = \arctan\left(\frac{2x+1}{\sqrt{7}}\right) + C_2.$$

$$x^2+x+2 = \left(x+\tfrac{1}{2}\right)^2 - \tfrac{1}{4} + 2 = \left(x+\tfrac{1}{2}\right)^2 + \frac{7}{4} = \frac{7}{4}\left[\frac{(x+1/2)^2}{7/4} + 1\right]$$

$$= \frac{7}{4}\left[\left(\frac{x+1/2}{\frac{\sqrt{7}}{2}}\right)^2 + 1\right] = \frac{7}{4}\left[\left(\frac{2x+1}{\sqrt{7}}\right)^2 + 1\right]$$

$$\int \frac{2x+1}{x^2+x+2}\,dx = \ln|x^2+x+2| + \arctan\left(\frac{2x+1}{\sqrt{7}}\right) + C$$