

TD N°3 : Simplification des fonctions

Exercice N°1 : Simplifier les équations suivantes :

$$F_1 = a.b + \bar{c} + c.(a + \bar{b})$$

$$F_2 = (x.\bar{y} + z).(x + \bar{y}).z$$

$$F_3 = (x + y).z + \bar{x}.(y + z) + \bar{y}$$

$$F_4 = (a + b + c).(a + b + c) + a.b + b.c$$

Exercice N°2 : Établir les tables de vérité des fonctions suivantes, puis les écrire sous les deux formes canoniques :

<p>1. <math>F_1 = XY + IZ + XZ</math></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>X</th><th>Y</th><th>Z</th><th><math>F_1</math></th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>1</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td><td>1</td></tr> </tbody> </table> <ul style="list-style-type: none"> <li>• Première forme canonique</li> </ul> $F_1 = \bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$ <ul style="list-style-type: none"> <li>• Seconde forme canonique</li> </ul> $F_1 = (X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+Y+Z)$	X	Y	Z	$F_1$	0	0	0	0	0	0	1	0	0	1	0	0	0	1	1	1	1	0	0	0	1	0	1	1	1	1	0	1	1	1	1	1	<p>2. <math>F_2 = X+YZ + \bar{Y}\bar{Z}T</math></p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th>X</th><th>Y</th><th>Z</th><th>T</th><th><math>F_2</math></th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr> <tr><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr> </tbody> </table> <ul style="list-style-type: none"> <li>• Première forme canonique</li> </ul> $F_2 = \bar{X}\bar{Y}\bar{Z}T + \bar{X}\bar{Y}Z\bar{T} + \bar{X}YZT + X\bar{Y}\bar{Z}\bar{T} + X\bar{Y}\bar{Z}T + X\bar{Y}Z\bar{T} + X\bar{Y}ZT + XY\bar{Z}\bar{T} + XY\bar{Z}T + XYZ\bar{T} + XYZT$ <ul style="list-style-type: none"> <li>• Seconde forme canonique</li> </ul> $F_2 = (X+Y+Z+T)(X+Y+\bar{Z}+T)(X+Y+\bar{Z}+\bar{T})(X+\bar{Y}+Z+T)(X+\bar{Y}+Z+\bar{T})$	X	Y	Z	T	$F_2$	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	1	1	0	0	1	0	0	0	0	1	0	1	0	0	1	1	0	1	0	1	1	1	1	1	0	0	0	1	1	0	0	1	1	1	0	1	0	1	1	0	1	1	1	1	1	0	0	1	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1
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5.  $F_5 = (\bar{X}Y + X\bar{Y})\bar{Z} + (\bar{X}\bar{Y} + XY)Z$

X	Y	Z	F <sub>5</sub>
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

- Première forme canonique

$$F_5 = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ$$

- Seconde forme canonique

$$F_5 = (X+Y+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+Y+\bar{Z})(\bar{X}+\bar{Y}+Z)$$

6.  $F_6 = \bar{X} + YZ$

X	Y	Z	F <sub>6</sub>
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- Première forme canonique

$$F_6 = \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + \bar{X}YZ + XYZ$$

- Seconde forme canonique

$$F_6 = (\bar{X}+Y+Z)(\bar{X}+Y+\bar{Z})(\bar{X}+\bar{Y}+Z)$$

7.  $F_7 = \bar{X}\bar{Y}Z + X\bar{Y}\bar{Z} + X\bar{Y}Z + XY\bar{Z} + XYZ$

X	Y	Z	F <sub>7</sub>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- Première forme canonique : c'est la forme de l'énoncé.

$$F_7 = \bar{X}\bar{Y}Z + X\bar{Y}\bar{Z} + X\bar{Y}Z + XY\bar{Z} + XYZ$$

- Seconde forme canonique

$$F_7 = (X+Y+Z)(X+\bar{Y}+Z)(X+\bar{Y}+\bar{Z})$$

8.  $F_8 = (\bar{X}+\bar{Y}+Z)(X+\bar{Y}+Z)(X+\bar{Y}+\bar{Z})(X+Y+\bar{Z})(X+Y+Z)$

X	Y	Z	F <sub>8</sub>
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

- Première forme canonique

$$F_8 = X\bar{Y}\bar{Z} + X\bar{Y}Z + XYZ$$

- Seconde forme canonique : c'est la forme de l'énoncé.

$$F_8 = (X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+\bar{Y}+Z)$$

**Exercice N°3 : Complémenter** les expressions suivantes (sans simplification) :

1.  $F_1 = \bar{X}\bar{Y} + XY + \bar{X}Y$     4.  $F_4 = X\bar{Y}Z\bar{T} + \bar{X}YT + \bar{X}\bar{Z} + (Z+T)(X\bar{Y}+Z)$

2.  $F_2 = X(\bar{Y}\bar{Z} + YZ) + \bar{X}YZ + \bar{X}\bar{Y}Z$     3.  $F_3 = X\bar{Y} + Z\bar{T} + \bar{X}\bar{Y} + \bar{Z}\bar{T}$

Solution :

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1.  $\bar{F}_1 = (X + Y)(\bar{X} + \bar{Y})(X + \bar{Y})$
2.  $\bar{F}_2 = (\bar{X} + (Y + Z)(\bar{Y} + \bar{Z}))(X + \bar{Y} + Z)(X + Y + \bar{Z})$
3.  $\bar{F}_3 = (\bar{X} + Y)(\bar{Z} + T)(X + Y)(Z + T)$
4.  $\bar{F}_4 = (\bar{X} + Y + \bar{Z} + T)(X + \bar{Y} + \bar{T})(X + Z)(\bar{Z}\bar{T} + (\bar{X} + Y) \cdot \bar{Z})$

**Exercice N°4 :** Écrire sous la première forme canonique les fonctions définies par les propositions suivantes :

1.  $f(A, B, C) = 1$  si et seulement si aucune des variables  $A, B, C$  ne prend la valeur 1
2.  $f(A, B, C) = 1$  si et seulement si au plus une des variables  $A, B, C$  prend la valeur 0
3.  $f(A, B, C) = 1$  si et seulement si exactement une des variables  $A, B, C$  prend la valeur 1
4.  $f(A, B, C) = 1$  si et seulement si au moins l'une des variables  $A, B, C$  prend la valeur 0
5.  $f(A, B, C) = 1$  si et seulement si exactement deux des variables  $A, B, C$  prennent la valeur 1
6.  $f(A, B, C) = 1$  si et seulement si au moins deux des variables  $A, B, C$  prennent la valeur 0
7.  $f(A, B, C) = 1$  si et seulement si les variables  $A, B, C$  prennent la valeur 1

Solution :

Utiliser les combinaisons des variables pour lesquelles  $f = 1$ .

1.  $f(A, B, C) = \bar{A}\bar{B}\bar{C}$
2.  $f(A, B, C) = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$
3.  $f(A, B, C) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$
4.  $f(A, B, C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$
5.  $f(A, B, C) = \bar{A}BC + A\bar{B}C + AB\bar{C}$
6.  $f(A, B, C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C}$
7.  $f(A, B, C) = ABC$

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Exercice N° 5 :

Trouvez les équations des tables de vérité de S, T et U avec les variables A, B, C et D :

A	B	C	D	S	T	U
0	0	0	0	0	1	0
0	0	0	1	0	1	1
0	0	1	0	0	0	1
0	0	1	1	0	1	1
0	1	0	0	0	0	0
0	1	0	1	1	1	1
0	1	1	0	1	1	0
0	1	1	1	1	1	0
1	0	0	0	0	0	0
1	0	0	1	1	1	1
1	0	1	0	0	1	1
1	0	1	1	1	1	1
1	1	0	0	0	1	0
1	1	0	1	0	1	1
1	1	1	0	1	1	0
1	1	1	1	1	1	0

Exercice N° 6 : Démontrer les relations suivantes

- $AB + ACD + \overline{B}D = AB + \overline{B}D$
- $(\overline{A} + B)(A + C)(B + C) = (\overline{A} + B)(A + C)$
- $AB + \overline{B}C = (A + \overline{B})(B + C)$
- $\overline{AB} + \overline{AB} = AB + \overline{A}\overline{B}$
- $\overline{(A + B)(\overline{A} + C)} = (A + \overline{B})(\overline{A} + \overline{C})$

Solution :

$$\begin{aligned}
 1. \quad AB + ACD + \overline{B}D &= AB + ACD(\overline{B} + B) + \overline{B}D = AB + ABCD + A\overline{B}CD + \overline{B}D \\
 &= AB \underbrace{(1 + CD)}_{=1} + \overline{B}D \underbrace{(1 + AC)}_{=1} = AB + \overline{B}D \\
 2. \quad (\overline{A} + B)(A + C)(B + C) &= (\overline{A} + B)(A + C)(B + C + \overline{A}A) = (\overline{A} + B)(A + C)(B + C + \overline{A})(B + C + A) \\
 &= (\overline{A} + B + \underbrace{0}_{=0}C)(A + C + \underbrace{0}_{=0}B) = (\overline{A} + B)(A + C) \\
 3. \quad AB + \overline{B}C &= AB \underbrace{(1 + C)}_{=1} + \overline{B}C \underbrace{(1 + A)}_{=1} = AB + \overline{B}C + ABC + A\overline{B}C = AB + \overline{B}C + AC \\
 &= AB + \underbrace{\overline{B}B}_{=0} + \overline{B}C + AC = (A + \overline{B})B + (A + \overline{B})C = (A + \overline{B})(B + C) \\
 4. \quad \overline{AB} + \overline{AB} &= \overline{AB} \cdot \overline{AB} = (\overline{A} + B)(\overline{A} + \overline{B}) = \underbrace{\overline{A}A}_{=0} + \overline{A}\overline{B} + BA + \underbrace{B\overline{B}}_{=0} = AB + \overline{A}\overline{B} \\
 5. \quad \overline{(A + B)(\overline{A} + C)} &= \overline{A + B} + \overline{\overline{A} + C} = (\overline{A} \cdot \overline{B}) + (A \cdot \overline{C}) = \underbrace{(\overline{A} + A)}_{=1}(\overline{A} + \overline{C})(\overline{B} + A)(\overline{B} + \overline{C}) \\
 &= (\overline{A} + \overline{C})(A + \overline{B})(\underbrace{\overline{A}A}_{=0} + \overline{B} + \overline{C}) = (\overline{A} + \overline{C})(A + \overline{B})(\overline{A} + \overline{B} + \overline{C})(A + \overline{B} + \overline{C}) \\
 &= (\overline{A} + \underbrace{0}_{=0}\overline{B} + \overline{C})(A + \overline{B} + \underbrace{0}_{=0}\overline{C}) = (A + \overline{B})(\overline{A} + \overline{C})
 \end{aligned}$$