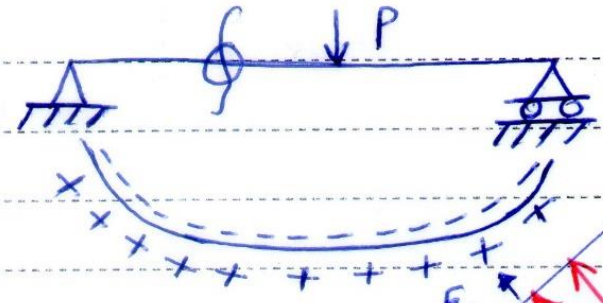
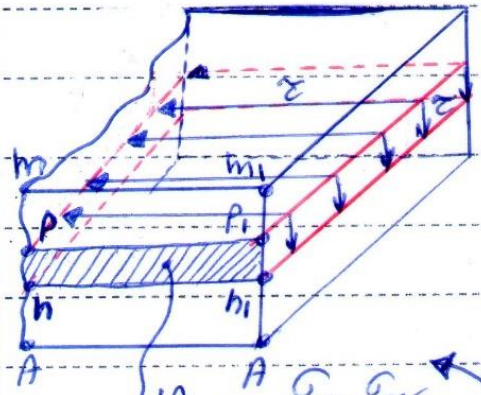
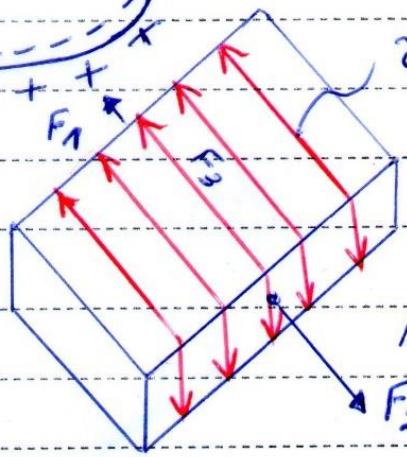
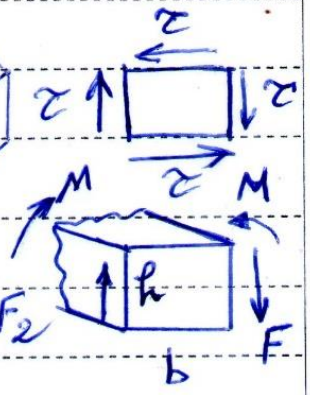
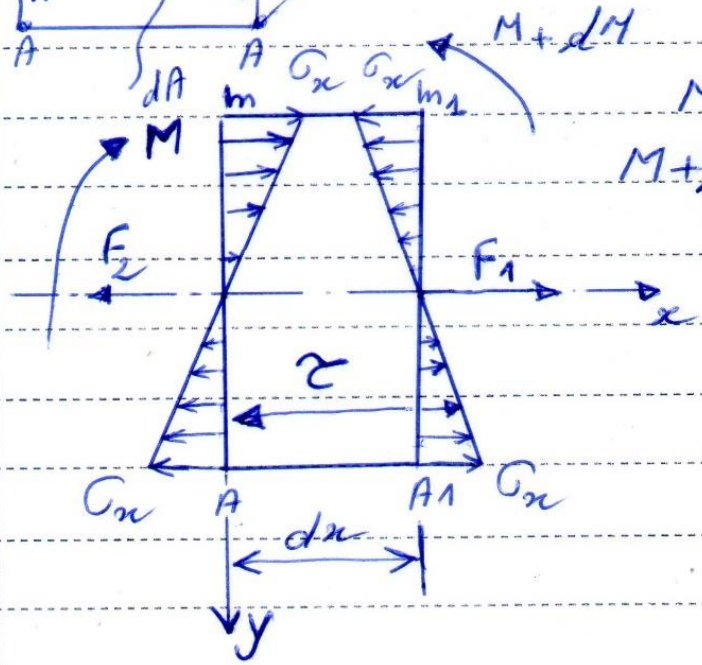
* Contraintes tangentielles dans les poutres :
الاجهادات المماسية في العتبات



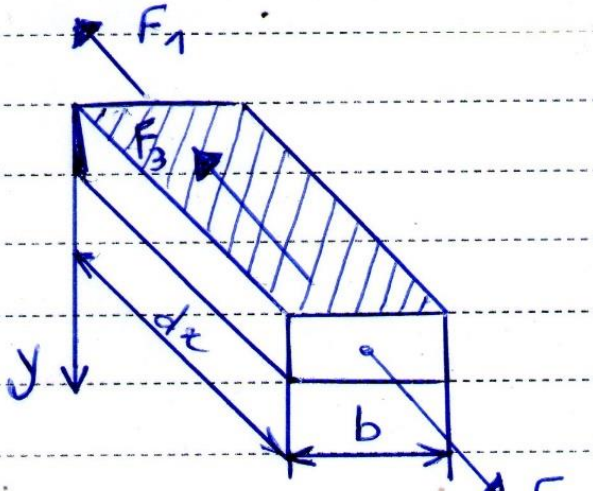








* M ← المقطع $(m\,n)$
* $M+dM$ ← المقطع $(m_1\,n_1)$



* $dF = \sigma_x \cdot dA = \dfrac{M\,y}{I} \cdot dA$

$$F_1 = \int \frac{M\,y}{I} \cdot dA \longrightarrow (m,n) \qquad F_2 = \int \frac{(M+dM)}{I} \cdot dA \longrightarrow (m_1,n_1)$$

* $F_3 = \tau \cdot b \cdot dx$

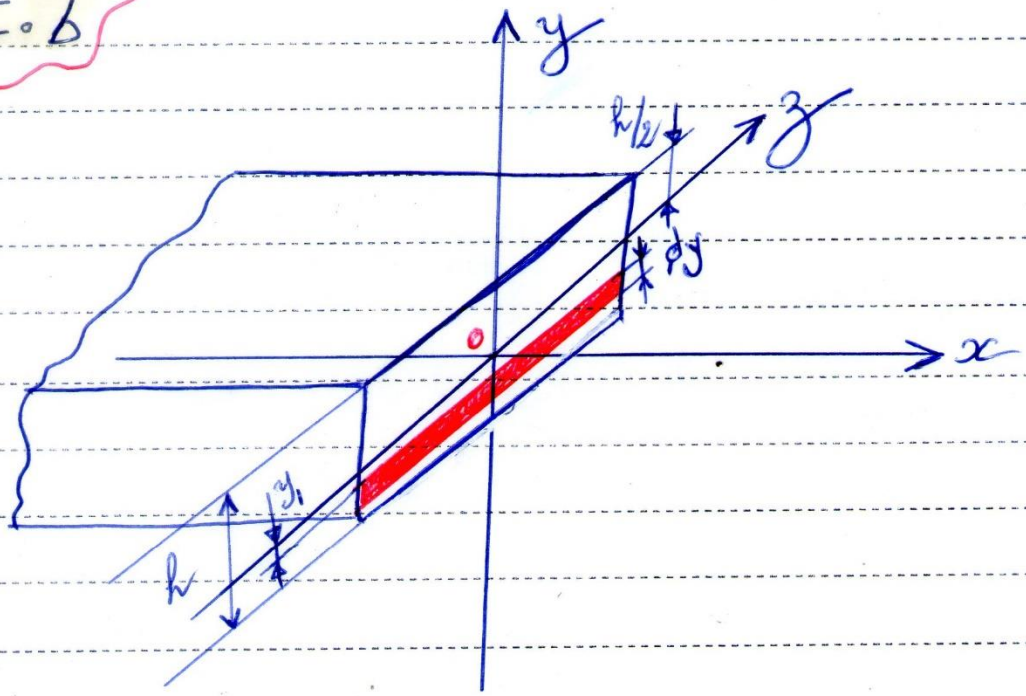
الجسم في توازن $\Rightarrow \sum F=0 \Rightarrow F_2 - F_1 - F_3 = 0$

$$\tau \cdot b \cdot dx = \int \frac{M \cdot y \cdot dA}{I} + \frac{dM \cdot y}{I} \cdot dA - \int \frac{M y}{I} \cdot dA$$

$$\tau \cdot b \cdot dx = \int \frac{dM}{I} \cdot y \Rightarrow \tau = \frac{dM}{dx}\left(\frac{1}{I \cdot b}\right) \int_A y \cdot dA$$

$$\tau = \frac{F}{I \cdot b} \int_A y \cdot dA \quad \text{moment statique}$$

$$\tau = \frac{F \cdot S}{I \cdot b}$$

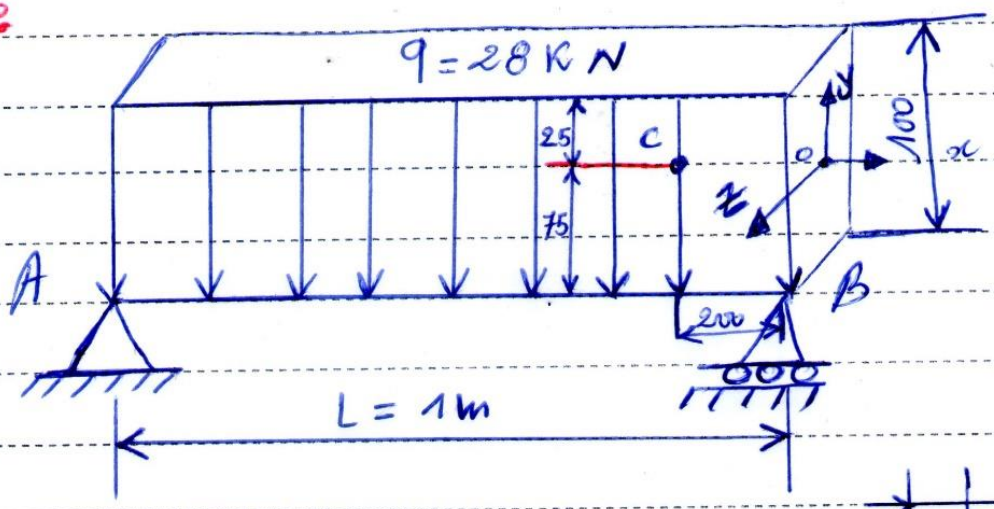


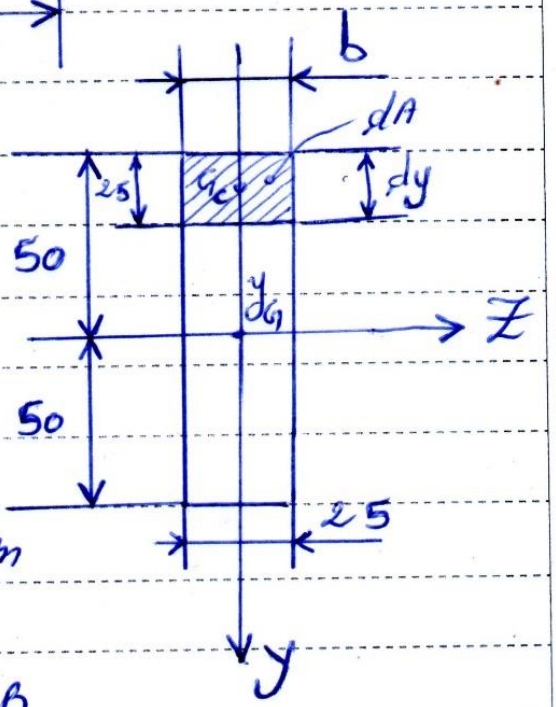
$I = I_i + (y_G - y_i)^2 \cdot A_i$, $dA = dy \cdot b$

$$S = \int_A y \cdot dA = \int_{y_1}^{h/2} y \cdot (b \cdot dy) = b \int_{y_1}^{h/2} y \cdot dy = b \left[\frac{y^2}{2}\right]_{y_1}^{h/2}$$

$$S = \frac{b}{2}\left[\left(\frac{h}{2}\right)^2 - y_1^2\right] = \frac{b}{2}\left[h^2/4 - y_1^2\right]$$

EX01:



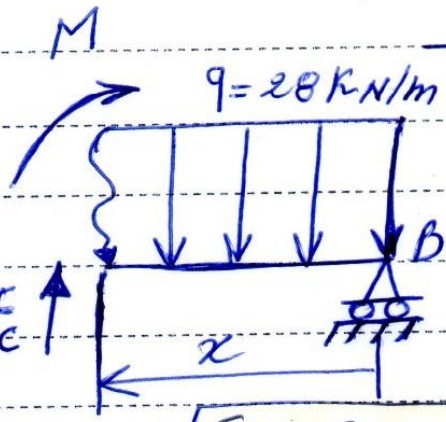


$R_A + R_B = 28$

$\sum M/_B = 0 \Rightarrow -R_A \cdot 1 + 28 \cdot \frac{1}{2} = 0$

$\Rightarrow \boxed{R_A = 14\ KN}$

$\boxed{R_B = 14\ KN}$



$F + R_B - 28x = 0 \Rightarrow \boxed{F = -R_B + 28x}$

$+ M - R_B \cdot x + 28\frac{x^2}{2} = 0 \Rightarrow \boxed{M = R_B \cdot x - \frac{28 \cdot x^2}{2}}$

Le point C : $F_C = 28x + 14 = 28(200.10^{-3}) - 14$

$\boxed{F_C = -8,4\ KN}$; $M_c = 14x - \frac{28x^2}{2}$

$M_C = 14(200.10^{-3}) - 14(200.10^{-3})^2 = \boxed{2,24\ KN.m}$

$\sigma_x = \frac{MJ}{I} \Rightarrow \boxed{I = \frac{bh^3}{12}} \Rightarrow I = \frac{(25).10^{-3}.(100.10^{-3})^3}{12}$

$\boxed{I = 2,083\ m^4}$

$$\sigma_{x(C)} = \frac{2,24.10^{3}.(-25).10^{-3}}{2,083} \quad / \quad \sigma_{x(C)} = \frac{M_{(C)}.y_{(C)}}{I}$$

$$\boxed{\sigma_{x(C)} = -26,9\ MPa}$$ عند النقطة C: حساب اجهاد الانضغاط

* $\tau = ?$ $\quad \tau = \frac{F}{I.b} \int_{y_1}^{h/2} y.dy = \frac{F}{2I}\left[\frac{h^2}{4} - y_1^2\right]$

Le point $\underline{\underline{C}} \longrightarrow y_1(C) = -25.10^{-3}\ m$

$\tau = ?$ $\quad I = I_i + (y_G - y_i)^2.A_i$

$$I = I_1 + (y_G - y_1)^2.A_1 = \frac{(25.10^{-3}).(100.10^{-3})^3}{12} +$$

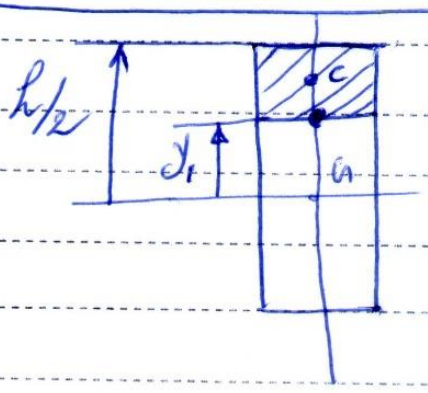
$$(50+12,5)^2.2500.10^{-6} \ldots\ldots$$

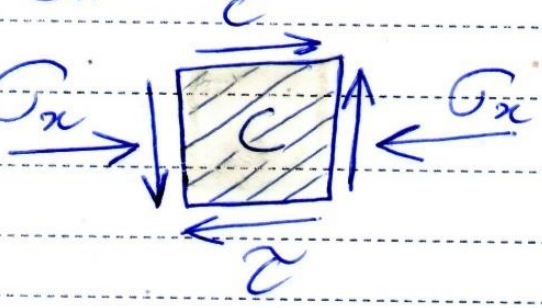
$$I = 3,515625.10^{-6} + 2,08333333.10^{-6}$$

$$\boxed{I = 5,59895833.10^{-6}\ m^4}$$

* Le point C $\longrightarrow y_1(C) = -25.10^{-3}\ m$

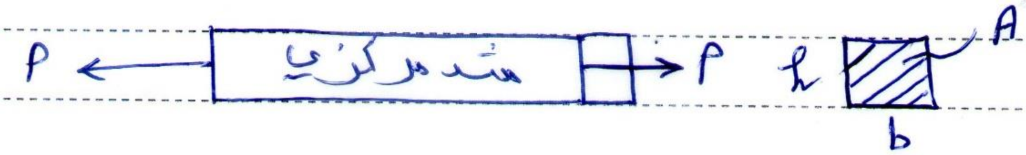
$$\boxed{\tau = -3,8\ MPa}$$

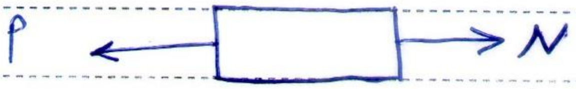




Traction ou compression excentrée

الشد او الانضغاط اللامركزي



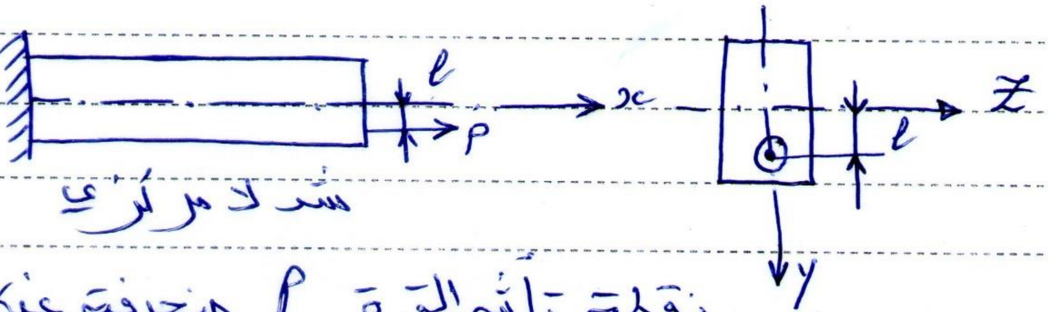


$\Rightarrow \sigma = \dfrac{P}{b \cdot h}$

$P = N$ (شد)

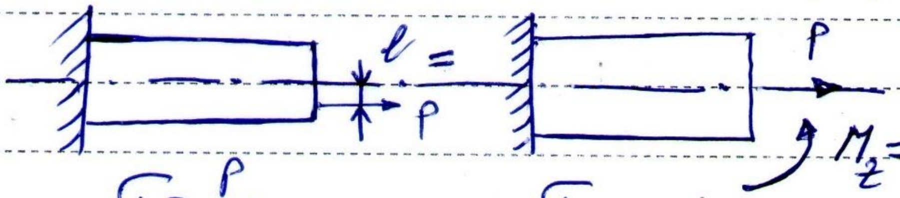
$P = -N$ (انضغاط)

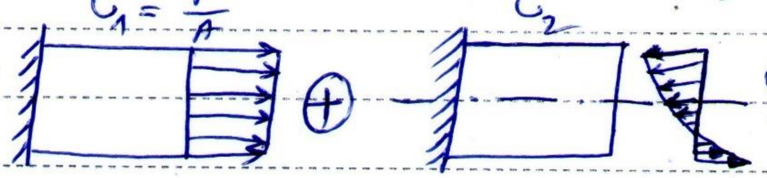
* $2^{ème}$ cas (Traction excentrée).



نقطة تأثير القوة P منحرفة عن محور واحد (Z).

- القوة P منحرفة عن محور يؤثر بصفة مكافئة للقوة P في مركز عزم إنحناء



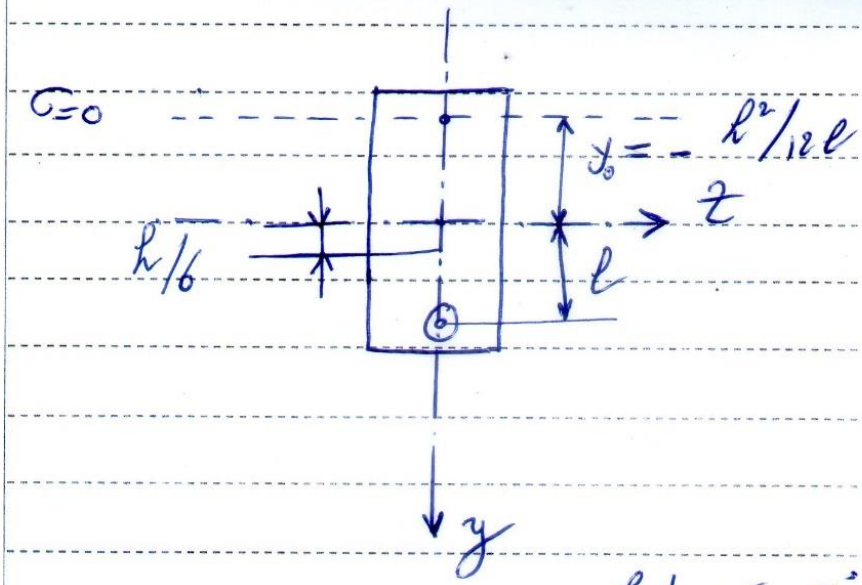


$\sigma_2 = \sigma_f = \dfrac{M y}{I} = \dfrac{P.e.y}{I}$

$\sigma = \sigma_1 + \sigma_2 = \dfrac{P}{A} + \dfrac{P.e}{I} \cdot y$

المحور المحايد: $\sigma = 0$

$\dfrac{P}{A} + \dfrac{P.e.y}{I} = 0 \Rightarrow y = \dfrac{-I}{A.e} \Rightarrow y = -\dfrac{bh^3/12}{b.h.e} = -\dfrac{h^2}{12e}$



* إجهاد الشد الأقصى: $y = h/2$

$$\sigma_{t_M} = \left(\frac{P}{b.h} + \frac{P.e.h/2}{\frac{bh^3}{12}}\right) = \left(\frac{P}{b.h} + \frac{P.e.h}{2} \cdot \frac{12}{bh^3}\right)$$

$$\sigma_{t_{Max}} = \left(\frac{P}{bh} + \frac{P.e.6}{bh^2}\right) = \frac{P}{bh}\left(1 + \frac{6.e}{h}\right)$$

* إجهاد الانضغاط الأقصى: $y = -h/2$

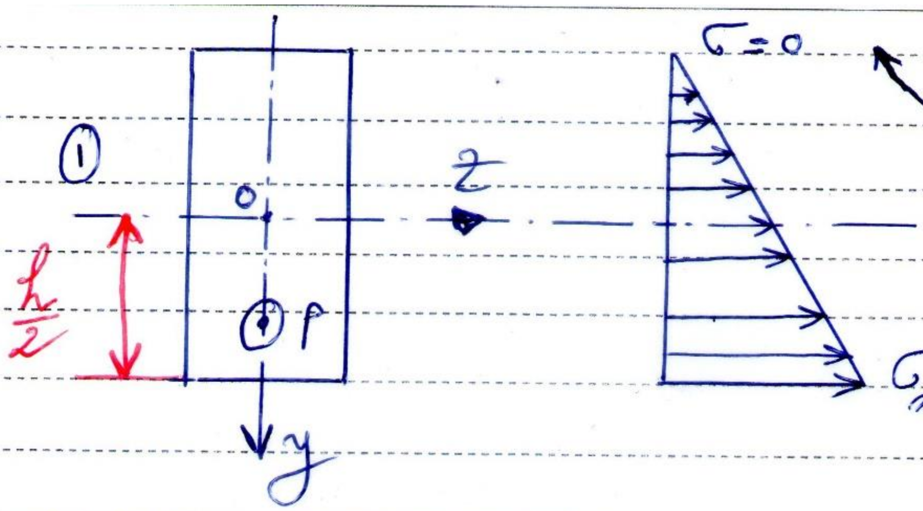
$$\sigma_{c_{max}} = \frac{P}{b.h}\left(1 - \frac{6.e}{h}\right)$$

$$1 - \frac{6.e}{h} = 0 \Rightarrow 1 = \frac{6.e}{h} \Rightarrow 6e = h \Rightarrow e = \frac{h}{6}$$

Remarque: $e = \frac{h}{2} \Rightarrow \sigma = 0$

العلوية ك
أي في الحناح المقطع $-h/2$
- إذا كانت $e < h/2$ ⇐ كل نقاط المقطع تكون تحت الشد
بقيم متفاوتة
- إذا كانت $e > \frac{h}{2}$ يكون جزء من المقطع العلوي تحت الانضغاط

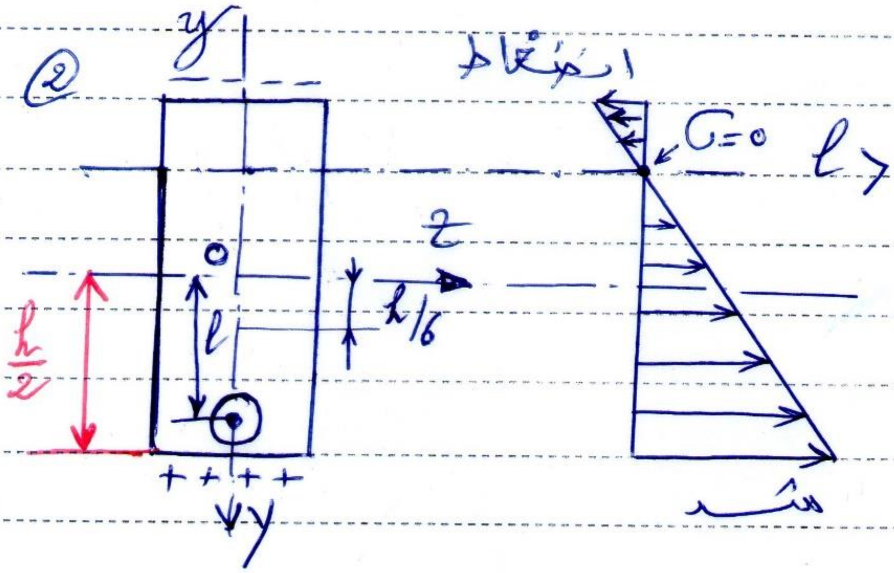
(1)



من اجل $l = h/2$

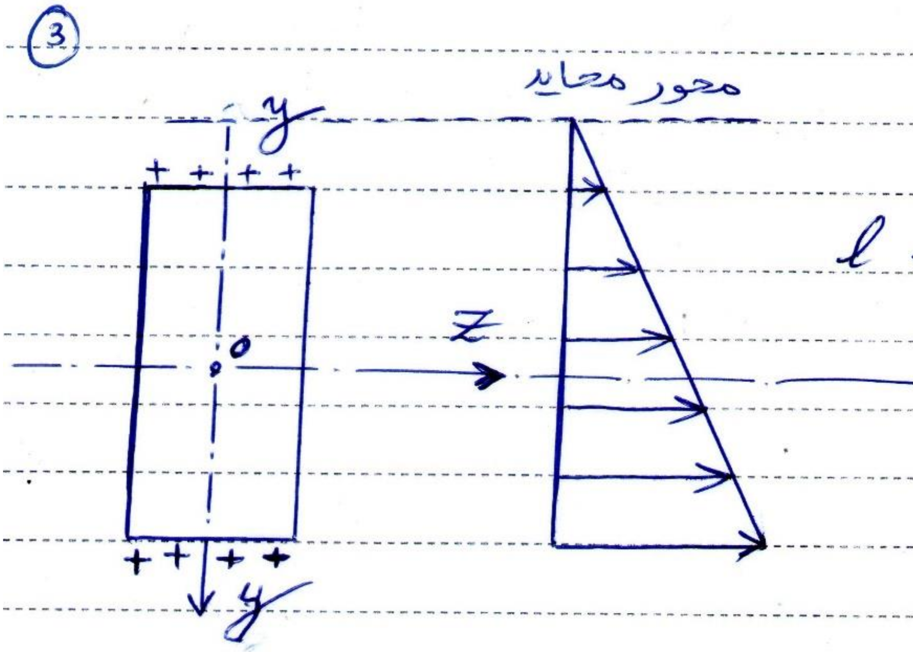
$$\sigma_{max} = \frac{P}{bh}\left(1 + \frac{6\left(\frac{h}{2}\right)}{h}\right)$$

(2)



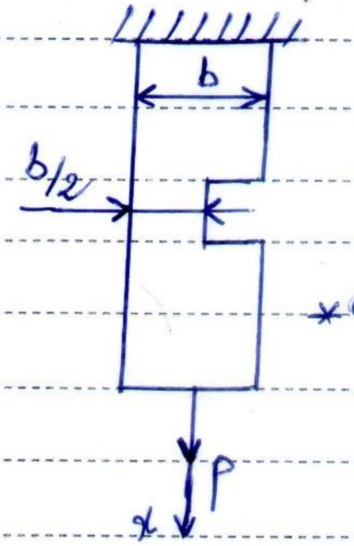
$l > h/2$

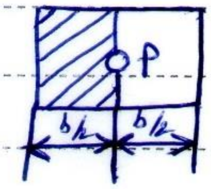
(3)



$l < h/2$

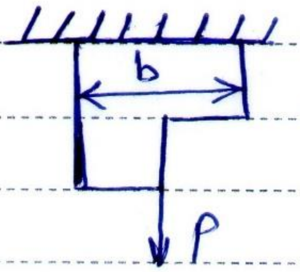
Ex 18 :



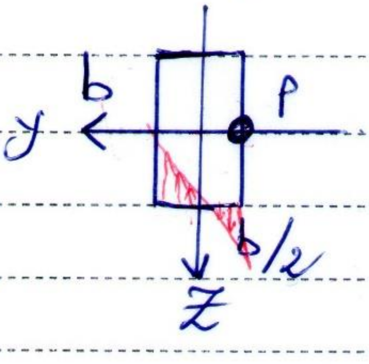


section AB

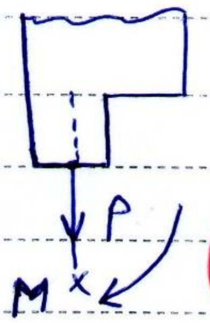
* calculer les contraintes dans la section mini



شد مركزي + انحناء



$\Rightarrow$



$$\sigma = \sigma_{Tra} + \sigma_{f}$$

$$\sigma = \frac{P}{A} + \frac{My}{I}$$

- pour $N = P$ (Traction) et pour ($N = -P$) (compression)

$M = P.\,b/4$ / $A = b\cdot\frac{b}{2} = b^2/2$, $I = \frac{b\cdot\left(\frac{b}{2}\right)^3}{12}$, $y_{max} = b/4$

* Nous remarquons que la poutre soumise un chargement de traction suit "x" et un moment fléchissant suit "z".

$$I = \frac{b^4}{96} \Rightarrow \sigma_{\substack{max \\ min}} = \frac{2P}{b^2} \pm \frac{96\left(\frac{P.b}{4}\cdot b/4\right)}{b^4}$$

$$\sigma_{\substack{max \\ min}} = \frac{2P}{b^2} \pm \frac{6P}{b^2} \Rightarrow \begin{cases} \sigma_{max} = \frac{8P}{b^2} \text{ (Traction)} \\ \sigma_{min} = \frac{-4P}{b^2} \text{ (compression)} \end{cases}$$

*