- Déterminée la position de l'axe neutre.

$$\sigma = 0 \Rightarrow \frac{N}{A} + \frac{My}{I} = 0 \Rightarrow \frac{-N}{A} = \frac{My_0}{I}$$

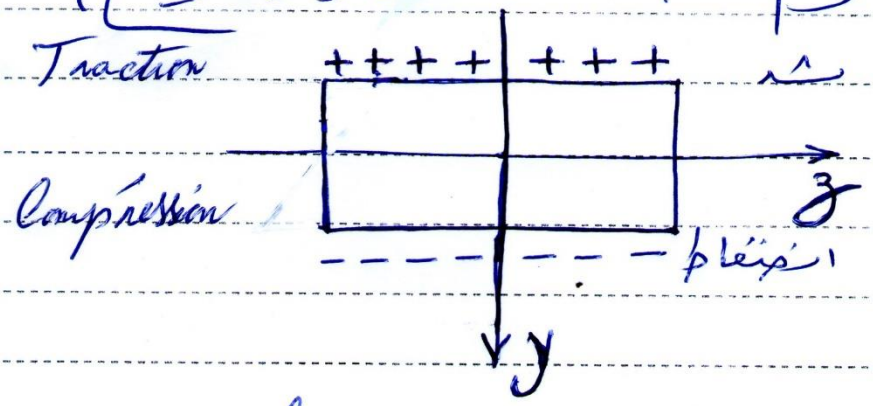
$$A.M.y_0 = -N.I \Rightarrow y_0 = \frac{A.M}{I.(-N)} = -\frac{A.M}{I.N}$$

$$y_0 = -\frac{N}{A}.\frac{I}{M} = -\frac{2P}{b^2}.\frac{b^4}{96}.\left(\frac{-4}{Pb}\right)$$

* Remarque : $M^{\oplus}$ (عزم موجب شد في الأعلى وانضغاط في الأسفل)

$M^{\ominus}$ (عزم سالب شد في الأسفل وانضغاط في الأعلى)

donc : $y_0 = \frac{b}{12}$



Exercice 02

$$\sigma_{max} = \frac{M_{max}}{I}.y_{max} = \frac{M_{max}.h/2}{I} = \frac{M.h/2}{\frac{bh^3}{12}} = \frac{P.a.\frac{h}{2}}{\frac{b.h^3}{12}}$$

$$\sigma = \frac{P.a.h.12}{2.bh^3} = \boxed{\frac{6.P.a}{b.h^2}} \Rightarrow \sigma_{max} \leq [\sigma]$$

$$\frac{6.P.a}{b.h^2} \leq 11 \Rightarrow \frac{3P}{2,25.10^{-3}} \leq 11.10^6 \Rightarrow \boxed{P = 8250\ N}$$

$$\tau_{max} \leq [\tau] \Rightarrow \frac{F.S}{I.b} \leq [\tau] \Rightarrow \frac{F.b.h^2}{8.I.b} \leq [\tau]$$

$$\frac{3.P}{2.b.h} \leq [\tau] \Rightarrow 3.P \leq 1,2.10^6.0,015 \Rightarrow P \leq 12000\ N$$

$$\Rightarrow P = \inf[8250]\ N$$

-2.1-

Exercice 06 : La poutre de la figure - 1 - elle soumis

② moments maximales (M_z et M_y).

$$\sigma_{f_z} + \sigma_{f_y} \leq \sigma_{max} \longrightarrow ①$$

$$① \Leftrightarrow \frac{M_z \cdot y_{max}}{I_z} + \frac{M_y \cdot z_{max}}{I_y} \leq \sigma_{max}$$

$$① \Leftrightarrow \frac{3P \cdot 0{,}3 \cdot 0{,}01 \cdot 12}{0{,}1 \cdot 0{,}2^3} + \frac{0{,}5 \cdot P \cdot 0{,}16 \cdot 0{,}005 \cdot 12}{0{,}02 \cdot 0{,}01^3} \leq 160$$

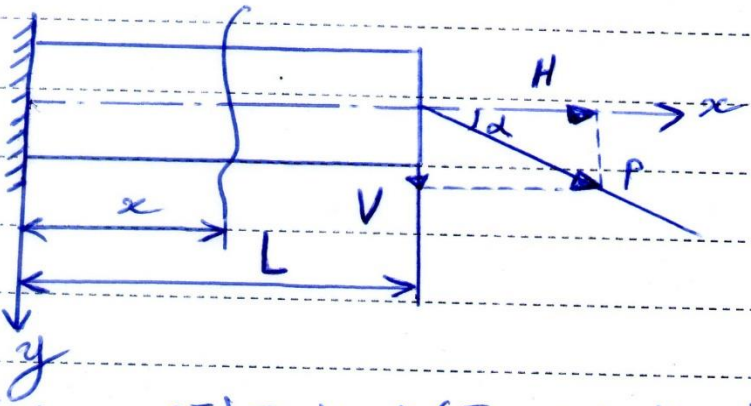
$$\Leftrightarrow \frac{2{,}16 \cdot 10^{-9} P + 0{,}144 \cdot 10^{-4} P}{1{,}6 \cdot 10^{-11}} \leq 160 \cdot 10^6$$

$$\Leftrightarrow 1{,}440216 \cdot 10^{-5} \cdot P \leq 2{,}56 \cdot 10^{-11}$$

$$\Leftrightarrow P \leq \frac{2{,}56 \cdot 10^{-3}}{1{,}440216 \cdot 10^{-5}} \leq \boxed{177{,}7511 \text{ N}}$$

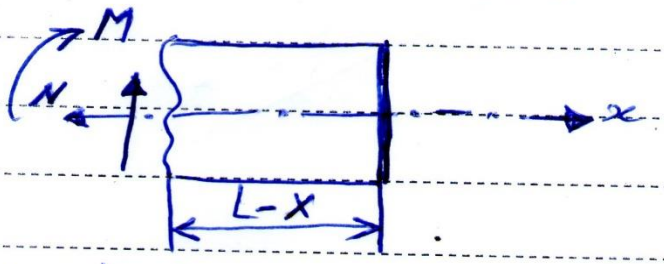
الانحناء مع الشد او مع الانضغاط

flexion avec traction ou avec compression



في كثير من الحالات تكون العتبات معرضة إلى الانحناء مع الشد او الانضغاط، فمثلا لما تكون القوة P مائلة فإنها المركبة الافقية "H" تحدث شد (Traction) بينما المركبة العمودية "V" تحدث انحناء "Flexion".

section ①:



$M = -V(L-x)$

$F = V$

$N = H$

- إذن في هذه الحالة الاجهاد الكلي يساوي:

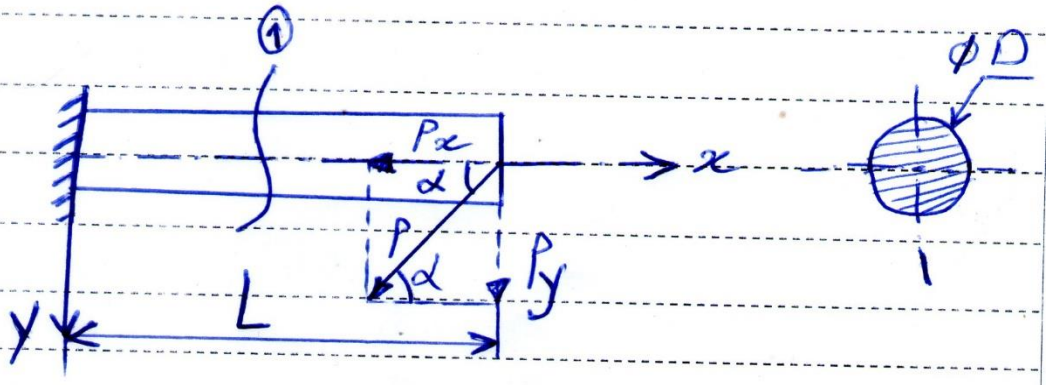
اجهاد الشد + اجهاد الانحناء

$\sigma = \sigma_1 + \sigma_2$

$\sigma = \frac{N}{A} + \frac{M}{I} \cdot y$

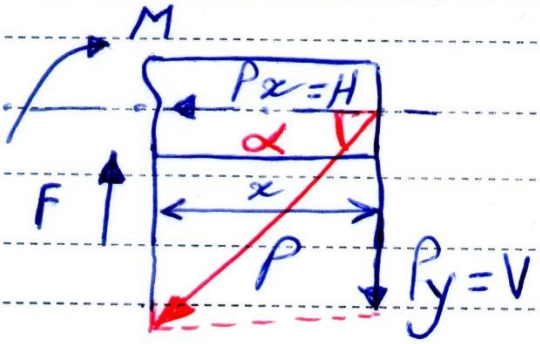
اجهاد + اجهاد

EX:



$L = 51\ cm,\ \alpha = 30°,\ D = 4\ cm,\ [\sigma_c] = -850\ MPa$

Section ①:



$\sigma = \sigma_1 + \sigma_2$

$H = P.\cos\alpha$

$V = P\sin\alpha$, $A = \frac{\pi.d^2}{4}$, $I = \frac{\pi.d^4}{64}$, $y_{max} = d/2$

$N = -P.\cos\alpha$

$F = V = P.\sin\alpha$, $M_{max} = -P.\sin\alpha.L$

$$\Rightarrow \sigma = \frac{N}{A} \pm \frac{M}{I}.y$$

$$\Rightarrow \sigma = \frac{-P.\cos\alpha.4}{\pi.d^2} \pm \frac{-P.\sin\alpha.L}{\frac{\pi.d^4}{64}}.\frac{d}{2}$$

$$\Rightarrow \sigma = \frac{-P.\cos\alpha.4}{\pi.d^2} - \frac{32.P.L.\sin\alpha}{\pi.d^3} \leq [\sigma_c]$$

$$\Rightarrow P\left[\frac{+4.\cos\alpha}{\pi.d^2} + \frac{32.L.\sin\alpha}{\pi.d^3}\right] \leq +850$$

$$\Rightarrow P\left[\frac{4.\cos 30^\circ}{\pi.(4.10^{-2})^2} + \frac{32.51.10^{-2}.\sin 30}{\pi.(4.10^{-2})^3}\right] \leq 850.10^6$$

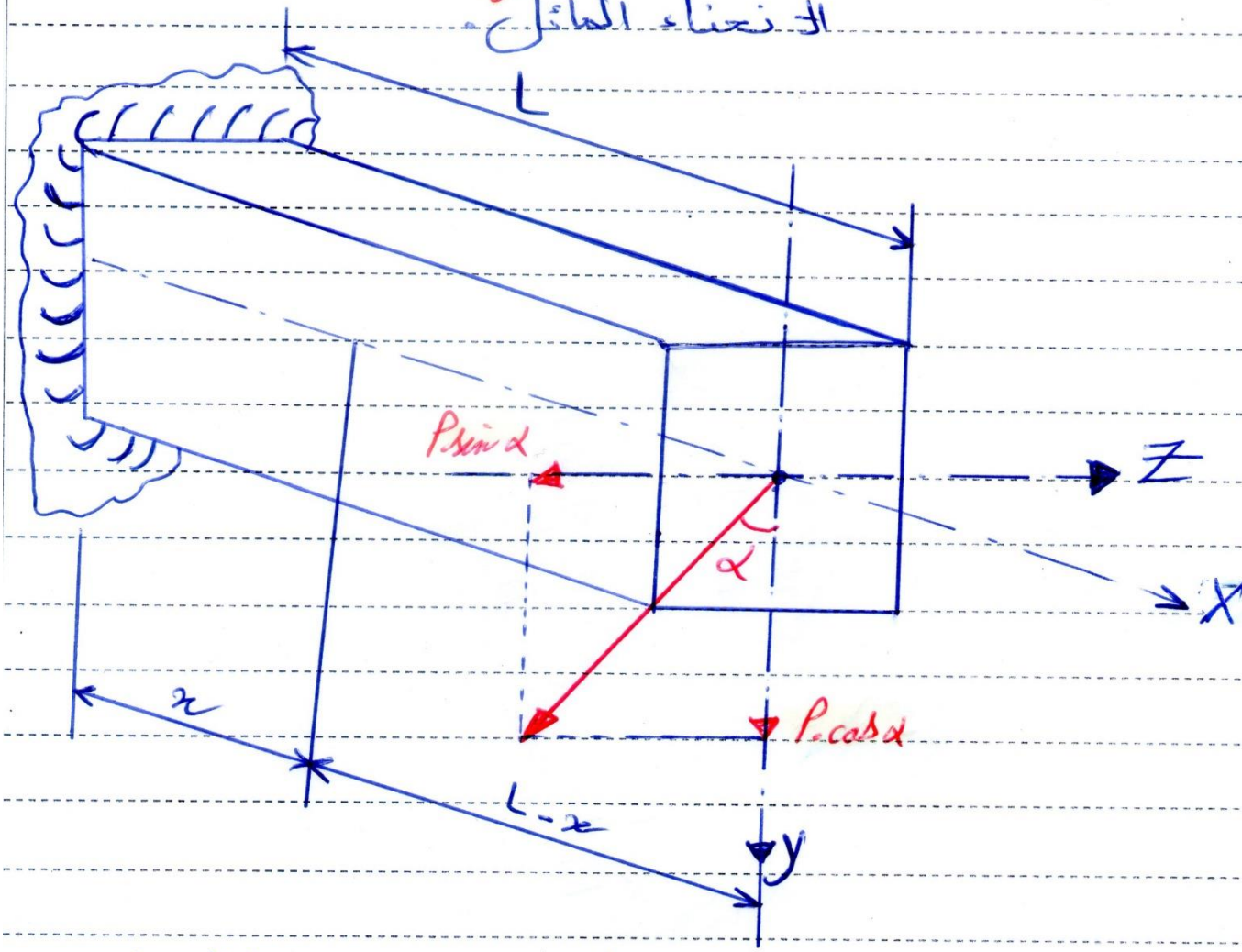
$$\Rightarrow P[689,5106 + 40605,09554] \leq 850.10^6$$

$$\Rightarrow P[2,823010153.10^{-5}] \leq 850$$

$$\Rightarrow P \leq 20583,80209\ Pa$$

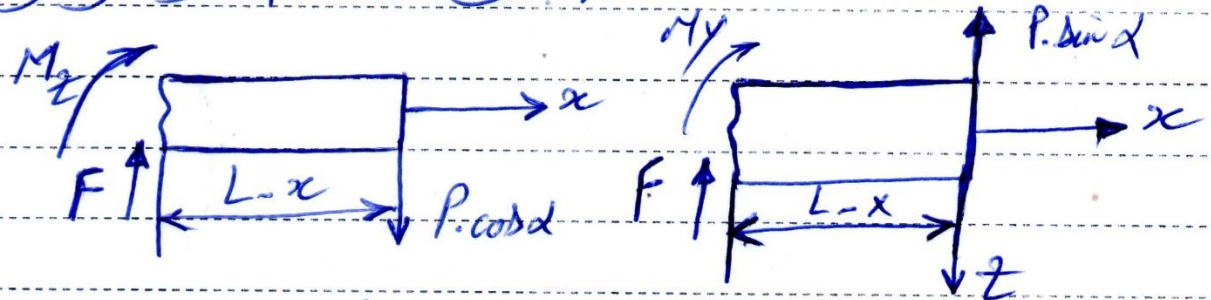
flexion déviée

الانحناء المائل



* في بعض الحالات تكون القوة المؤثرة على العتبات مائلة بالنسبة لمحاور المرتيسة المقطع.

* ندرس حالة عتبة تحت تأثير قوة P في نهايتها مائلة بالنسبة للمحور "Y" بزاوية "α" وتمر بمركز القطع.



$$M_Z = -P.\cos\alpha\,(L-x) \qquad M_y = P\sin\alpha\,(L-x)$$

$$* \sigma = \frac{M_{Y_0}}{I_Y} Z + \frac{M_Z}{I_Z} . Y$$

$$\sigma = \frac{M_y}{I_y} \cdot z + \frac{M_z}{I_z} \cdot y \Rightarrow \sigma = 0 \quad \text{(المحور المحايد)}$$

$$\sigma = 0 \Rightarrow \frac{M_y}{I_y} \cdot z + \frac{M_z}{I_z} \cdot y = 0 \Rightarrow \frac{M_y}{I_y} \cdot z = -\frac{M_z}{I_z} \cdot y$$

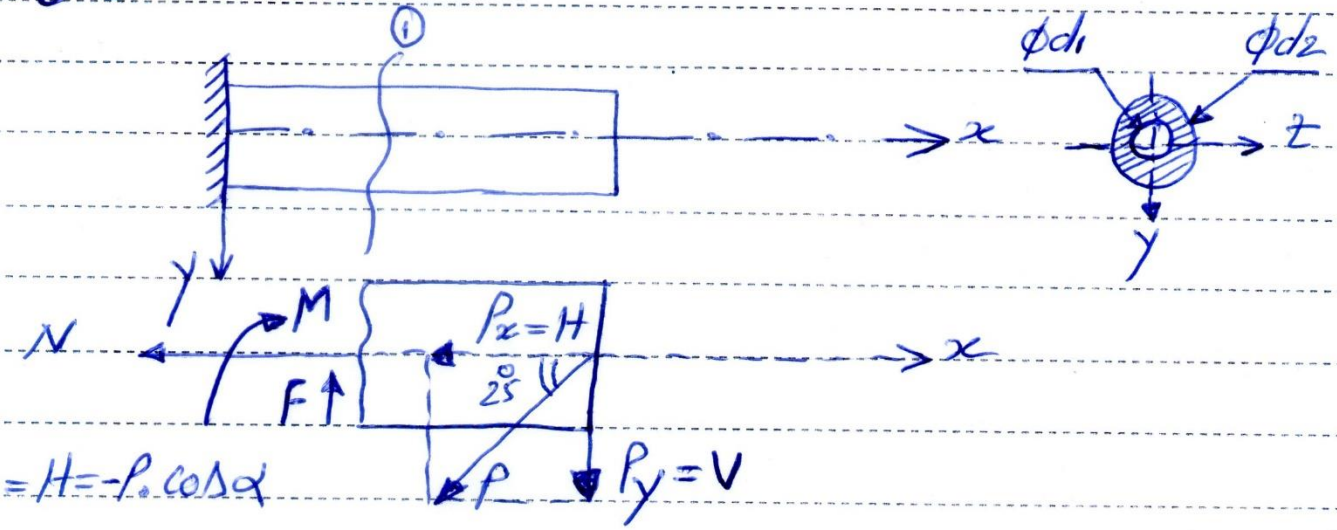
$$\frac{y}{z} = -\frac{M_y}{I_y} \cdot \frac{I_z}{M_z} = -\frac{M_y}{M_z} \cdot \frac{I_z}{I_y}, \quad tg\,\beta = \frac{y}{z}$$

$$tg\,\beta = \frac{y}{z} = \frac{-R \sin\alpha (L-x) \cdot I_z}{-R \cos\alpha (L-x) \cdot I_y}$$

$$tg\,\beta = \frac{y}{z} = tg\,\alpha \cdot \frac{I_z}{I_y}$$

EX n°4 $\alpha = 25°$; $L = 2m$, $d_1 = 200\,mm$, $d_2 = 260\,mm$

$[\sigma_c] = 900\,MN/m^2$



$P_x = H = -P.\cos\alpha$

$P_y = V = P.\sin\alpha$

$$A = \frac{\pi(d_2-d_1)^2}{4} = \frac{\pi(260.10^{-3} - 200.10^{-3})^2}{4} = \boxed{2,826\,10^{-3}\,m^2}$$

$F = V = P\sin\alpha$, $M = -V(L) = -P.\sin\alpha.L$

$N = -H = -P.\cos\alpha \Rightarrow \sigma = \sigma_1 + \sigma_2 = \frac{N}{A} + \frac{M}{I}.y$

$$\sigma = \frac{-P.\cos\alpha.4}{\pi.(d_2-d_1)^2} + \frac{-P.\sin\alpha.L}{\frac{\pi(d_2-d_1)^4}{32}} \cdot d_2/2$$

$$\sigma = \frac{-P.\cos\alpha.4}{\pi(d_2-d_1)^2} - \frac{P.\sin\alpha.L.d_2.32}{\pi(d_2-d_1)^4} \leq [\sigma_c]$$

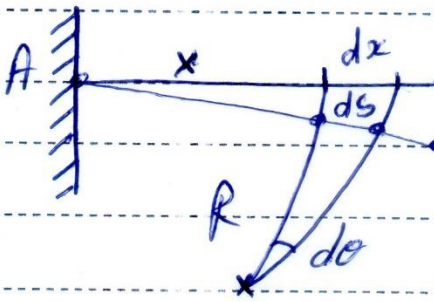
$$P\left[\frac{\cos\alpha.4}{\pi(d_2-d_1)^2} + \frac{\sin\alpha.L.d_2.16}{\pi(d_2-d_1)^4}\right] \leq \sigma_c$$

AN°

$$P\left[\frac{\cos 25.4}{\pi(0,26-0,2)^2} + \frac{\sin 25.2.0,26.16}{\pi(0,26-0,2)^4}\right] \leq 900.10^6$$

$$\boxed{P \leq 10377,59335\,N}$$

flexion déviée



AB: وضعية العتبة قبل التحميل

AB: // بعد التحميل ؛ منحنية.

$dx = ds$ عند طول من العتبة وهي منحنية يرمز (ds) عنصر قوس ، θ : زاوية دوران

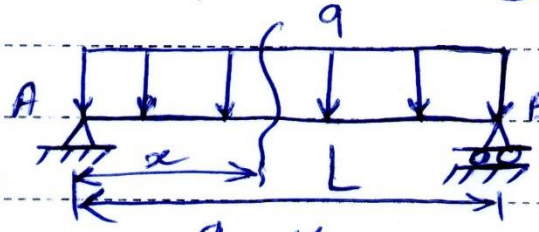
$* \; dx = ds, \; dx = R \cdot d\theta, \; \theta = \frac{dy}{dx}$

$$\Rightarrow \frac{d\theta}{dx} = \frac{1}{R} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} \Rightarrow \boxed{\frac{1}{R} = -\frac{M}{E.I}}$$

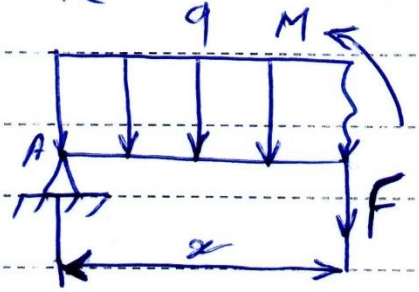
$$\Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{-M}{E.I}} \Rightarrow \boxed{EI\frac{d^2y}{dx^2} = -M}$$

معادلة تفاضلية لإزاحة العتبات.

EX:



$R_A = \frac{qL}{2}$ $\quad R_B = \frac{qL}{2}$



$R_A = \frac{qL}{2}$

$$\Rightarrow M - R_A \cdot x + q\frac{x^2}{2} = 0$$

$$\Rightarrow \boxed{M = qL/2 \cdot x - q \cdot \frac{x^2}{2}}$$

$$\boxed{EI \cdot \frac{d^2y}{dx^2} = \frac{q \cdot x^2}{2} - \frac{qL}{2} \cdot x}$$

$$\frac{EI\,d^2y}{dx^2} = q\cdot\frac{x^2}{2} - \frac{qL}{2}\cdot x$$

$$EI\frac{dy}{dx} = q\cdot\frac{x^3}{2\cdot 3} - \frac{qL\cdot x^2}{2\cdot 2} + C_1$$

$$EIy = \frac{q\,x^4}{4\cdot 6} - \frac{qL\cdot x^3}{4\cdot 3} + C_1\cdot x + C_2$$

* Les Conditions aux limites :

$x = 0 \Rightarrow y = 0 \Rightarrow \boxed{C_2 = 0}$ / Point "A"

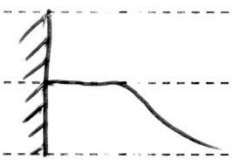
Point "B" : $x = L \Rightarrow y = 0 \Rightarrow \frac{qL^4}{24} - \frac{qL^4}{12} +$

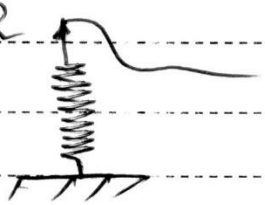
$C_1\cdot L + C_2 = 0 \Rightarrow C_1 = -\frac{qL^4}{12} + \frac{qL^4}{24}$ / L

$$C_1 = \frac{qL^4 - 2qL^4}{24\cdot L} = \boxed{\frac{-qL^3}{24}}$$

$$y = \frac{1}{EI}\left[\frac{q\cdot x^4}{24} - \frac{qL}{12}\cdot x^3 \quad \frac{qL^3}{24}x + 0\right]$$

* $\Rightarrow y = 0$

*  $\Rightarrow \frac{dy}{dx} = 0$

* 

$\Rightarrow y = \frac{R}{k}$