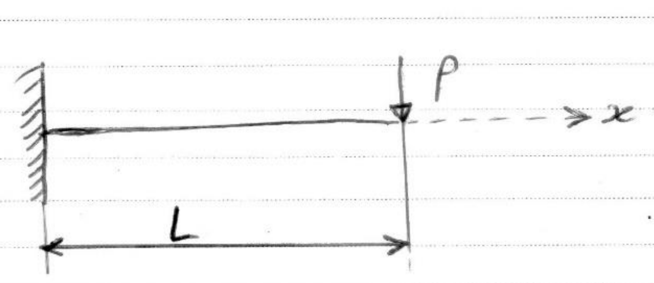
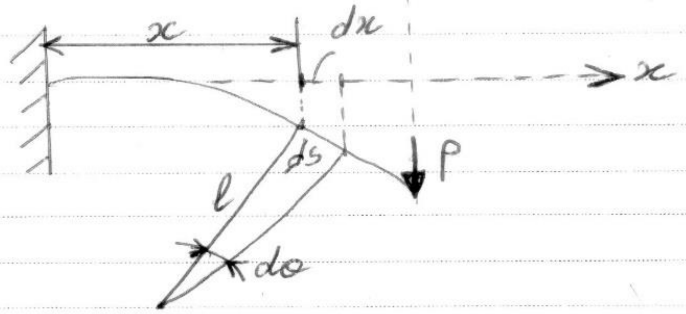
2 - Contraintes normales en flexion simple :

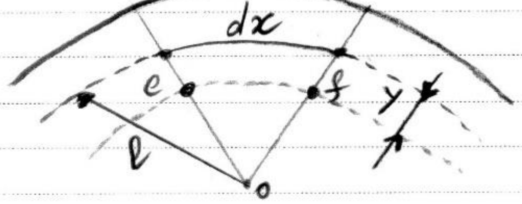




$\widehat{ds} = \ell . d\theta \Rightarrow ds = dx$

$\Rightarrow dx = \ell . d\theta \Rightarrow \dfrac{d\theta}{dx} = \dfrac{1}{\ell}$

$$\begin{cases} e.f = dx \\ ef = (\ell - y) d\theta \longrightarrow (1) \\ dx = \ell . d\theta \end{cases}$$



$(1) \Leftrightarrow ef = \ell\, d\theta - y\, d\theta = dx - y\, d\theta$

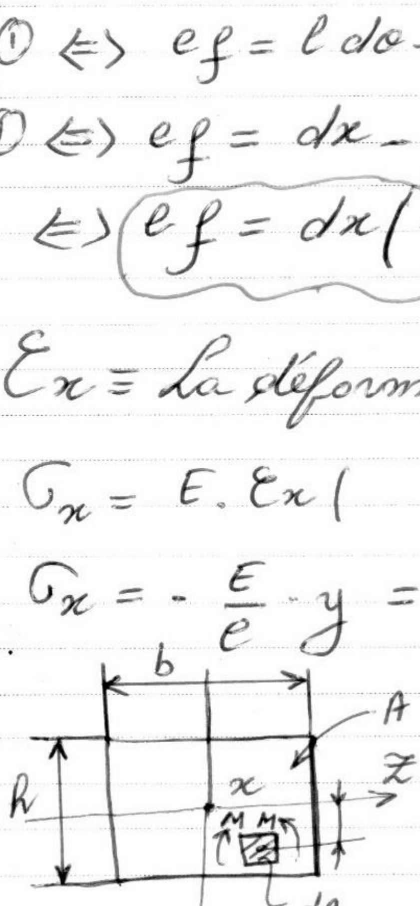
$(1) \Leftrightarrow ef = dx - y \dfrac{dx}{\ell} \Rightarrow ef = dx - \dfrac{y}{\ell} . dx$

$\Leftrightarrow ef = dx\left(1 - \dfrac{y}{\ell}\right)$ $\dfrac{y}{\ell} dx$ بقيمة ef تقليص طول الليفة *

$\mathcal{E}x$ = La déformation $\Rightarrow \mathcal{E}x = \dfrac{\text{التمدد او التقلص}}{\text{الطول الاصلي}} = \dfrac{-\frac{y}{\ell} . dx}{dx} = \dfrac{-y}{\ell}$

$\sigma_x = E . \mathcal{E}x$ (اجهاد الانحناء خاضع لقانون أوك) *

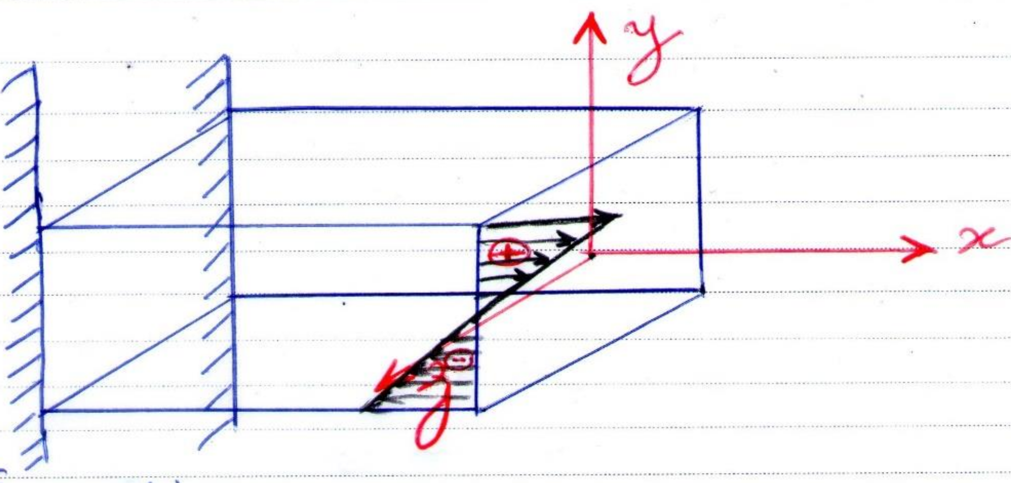
$\sigma_x = -\dfrac{E}{\ell} . y = E . \mathcal{E}x = E\left(\dfrac{-y}{\ell}\right) = \boxed{\dfrac{-E}{\ell} . y}$



$\Sigma F = 0 \Rightarrow \int_A \sigma_x . dA = 0 \Rightarrow \dfrac{-E}{\ell} \int_A . y . dA$

$\Rightarrow \int_A y . dA$ (moment statique de section A

ومنه المحور المحايد يمر بمركز الثقل المقطع A

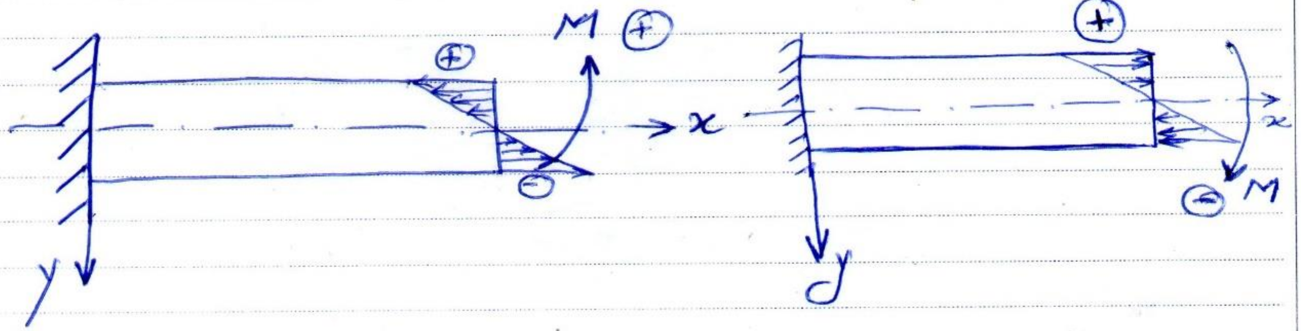


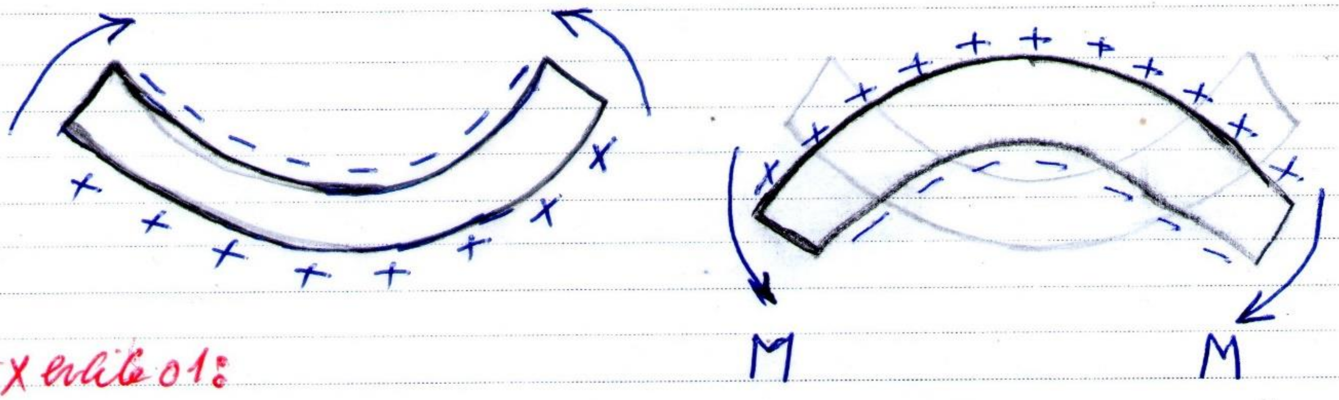
$$* \; M = \int_A \overbrace{\sigma_x \cdot dA}^{F} \cdot \overset{\text{ذراع}}{y} = \int_A \left(\frac{-E}{\rho}\right) \cdot y \cdot dA \cdot y$$

$$\Rightarrow M = -\frac{E}{\rho} \int_A y^2 \cdot dA \quad \text{moment d'inertie de } A.$$

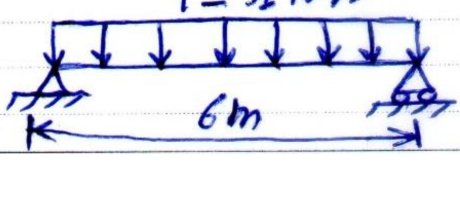
$$M = -\frac{E \cdot I}{\rho} \Rightarrow \frac{M}{I} = -\frac{E}{\rho} \Rightarrow \sigma_x = \frac{-E \cdot y}{\rho}$$

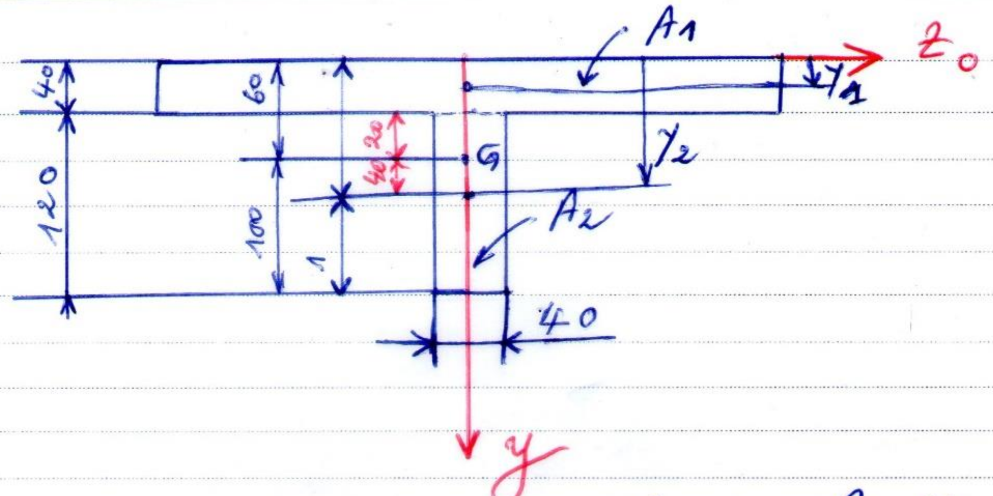
donc : $\boxed{\sigma_x = \frac{M}{I} y}$





EXercice 01 :





$$y_G = \Sigma \frac{A_i \cdot y_i}{\Sigma A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$y_G = \frac{(40 \cdot 120)\,20 + (40 \cdot 120)\,100}{(40 \cdot 120) + (120 \cdot 40)} = \boxed{60\,m}$$

$$* \boxed{I = \sum_i \left[I_i + (y_G - y_i)^2 A_i \right]}$$

$$I = \left[I_1 + (y_G - y_1)^2 A_1 \right] + \left[I_2 + (y_G - y_2)^2 \cdot A_2 \right]$$

$$I = \left[\frac{120 \cdot 40^3}{12} + (60 - 20)^2 (40 \cdot 120) \right] + \left[\frac{(40)(120)^3}{12} + (60 - 100)^2 \right.$$

$$\left. (120 \cdot 40) \right] = \boxed{21{,}76 \cdot 10^{-6}\ m^4}$$

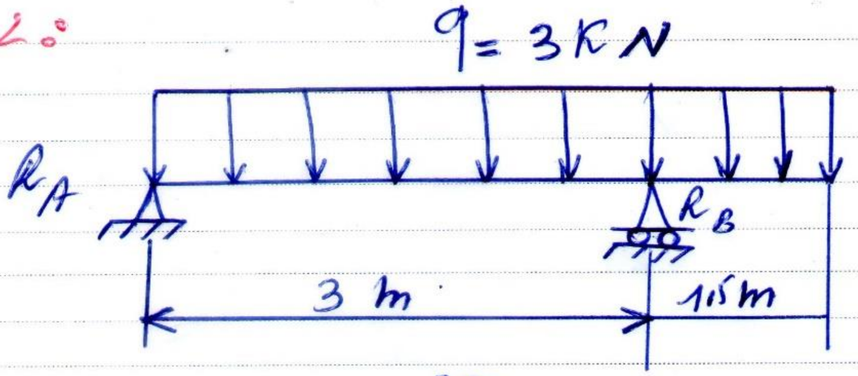
$$\sigma_x = \frac{MY}{I} \Rightarrow \sigma_{t\,max} = \frac{M_{max}\,\overline{Y_2}}{I}$$

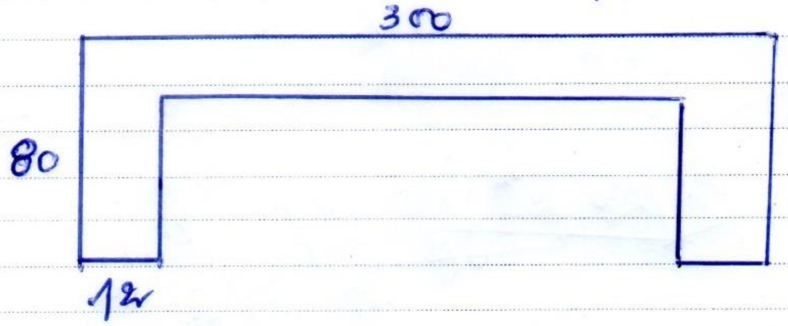
$$\sigma_{c\,max} = \frac{M_{max}\,\overline{Y_1}}{I}$$

$$\sigma_{t\,max} = \frac{4{,}5 \cdot 10^3 \cdot 100 \cdot 10^{-3}}{21{,}76 \cdot 10^{-6}} = \boxed{20{,}68\ Pa}$$

$$\sigma_{c\,max} = \frac{4{,}5 \cdot 10^3 \cdot 60 \cdot 10^{-3}}{21{,}76 \cdot 10^{-6}} = \boxed{12{,}41\ Pa}$$

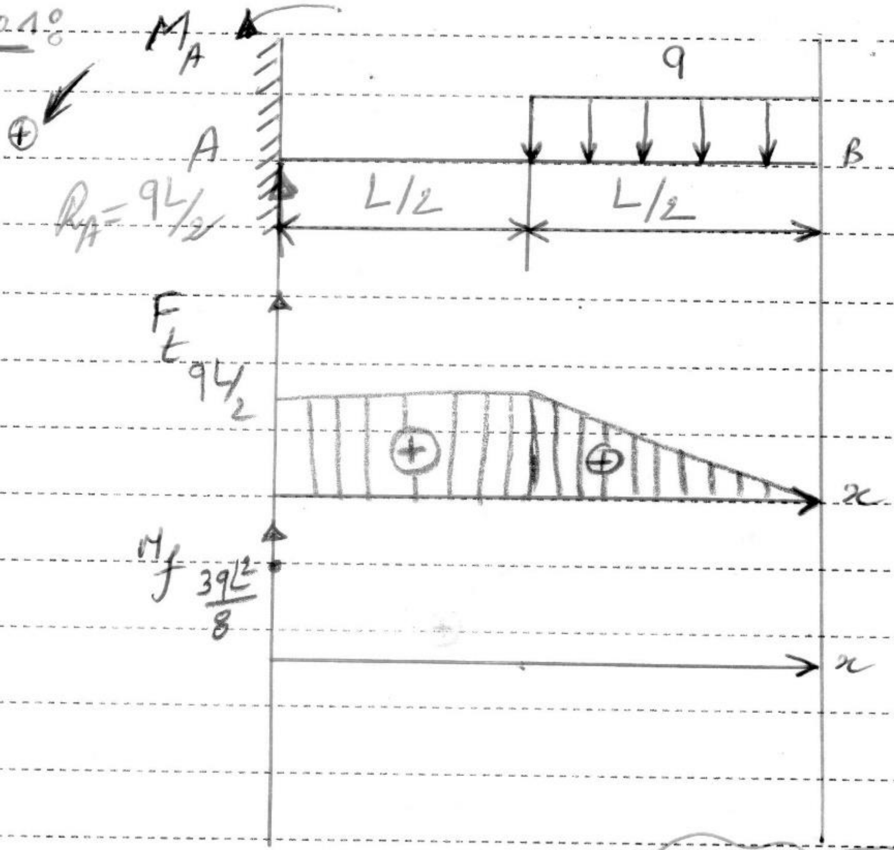
Exercice 02 :





* 1- Déterminée R_A et R_B
2- Diagramme de moment fléchissant.
3- Position de G
4- calculer de I
5- σ_t max et σ_c max

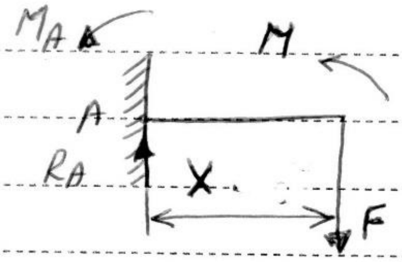
Exo 1°:



$$\Sigma F_{/x} = 0 \Rightarrow q \cdot \frac{L}{2} - R_A = 0 \Rightarrow R_A = q \cdot L/2$$

$$\Sigma M_{/A} = 0 \Rightarrow M_A + q \cdot \frac{L}{2}\left(\frac{L}{2} + \frac{L}{4}\right) = 0 \Rightarrow M_A = \frac{q \cdot 3L^2}{8}$$

$$\boxed{M_A = -3 \cdot qL^2/8}$$



* section ①: $0 \leq x \leq \frac{L}{2}$

$$\Sigma F = 0 \Rightarrow F = R_A = qL/2$$

$$\Sigma M_{/0} = 0 \Rightarrow M + M_A - R_A \cdot x = 0 \Rightarrow M = -M_A + qL/2 \cdot x$$

$$M = -\frac{3qL^2}{8} + qL/2 \cdot x$$

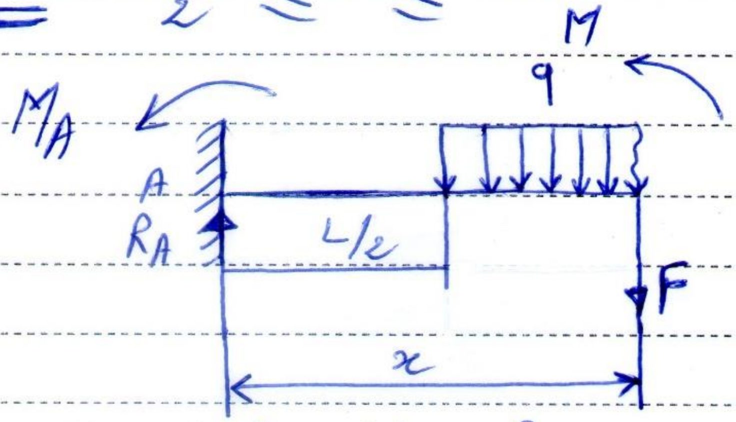
$$x = 0 \Rightarrow M = -3qL^2/8$$

$$x = L/2 \Rightarrow M = -\frac{3 \cdot qL^2}{8} + \frac{q \cdot L^2}{4}$$

$$x = \frac{L}{2} \Rightarrow M = \frac{-3 \cdot qL^2 + 2q \cdot L^2}{8}$$

$$\Rightarrow M = -qL^2/8$$

Section ②: $\frac{L}{2} \leq x \leq L$



$$\sum F = 0 \Rightarrow F + q\cdot\left(x - \frac{L}{2}\right) - R_A = 0$$

$$F = -q\left(x - \frac{L}{2}\right) + R_A = \boxed{-q\left(x - \frac{L}{2}\right) + qL/2}$$

$$* \; x = \frac{L}{2} \Rightarrow F = -q\left(\frac{L}{2} - \frac{L}{2}\right) + \frac{qL}{2}$$

$$\Rightarrow \boxed{F = \frac{qL}{2}}$$

$$* \; x = L \Rightarrow F = -\frac{qL}{2} + \frac{qL}{2} = \boxed{0}$$

$$* \; \sum_{/o} = 0 \Rightarrow M - R_A \cdot x + q\left(x - \frac{L}{2}\right)\cdot\left(\frac{L}{2} + \frac{x - L/2}{2}\right) = 0$$

$$M = \frac{qL}{2}\cdot x + q\left(x - \frac{L}{2}\right)\cdot\left(\frac{L}{2} + \frac{x - L/2}{2}\right)$$

$$x = \frac{L}{2} \Rightarrow M = \frac{qL^2}{4} + q\left(\frac{L}{2} - \frac{L}{2}\right)\left(\frac{L}{2} + \frac{x - L/2}{2}\right)$$

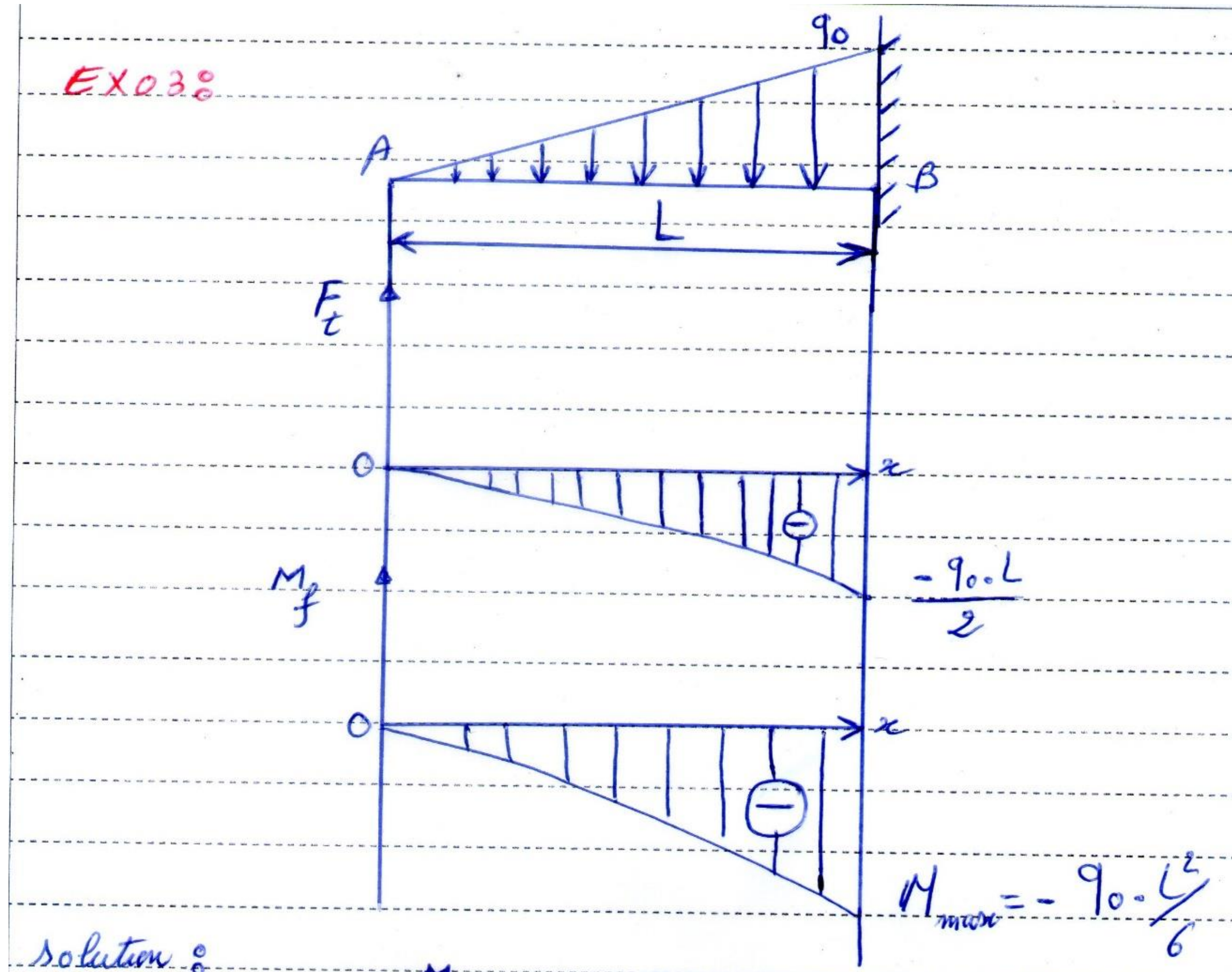
$$\Rightarrow M = \frac{qL^2}{2}$$

$$\cdot \; x = L \Rightarrow M = \frac{qL^2}{2} + \frac{qL}{2}\left(\frac{L}{2} + \frac{L}{4}\right)$$

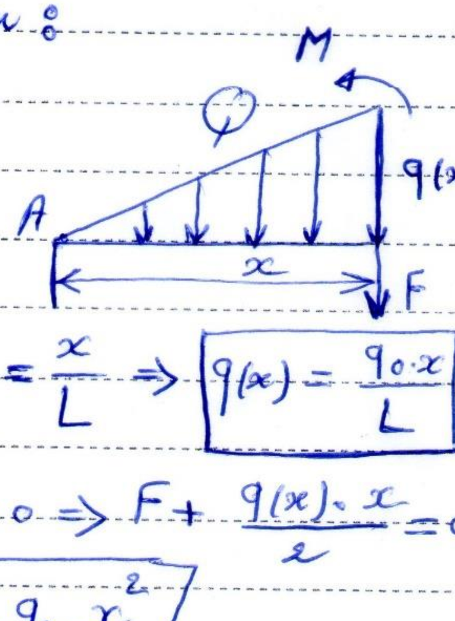
$$\Rightarrow M = \frac{qL^2}{2} + \frac{qL}{2}\left(\frac{2L + L}{4}\right)$$

$$\Rightarrow M = \frac{qL^2}{2} + \frac{3qL^2}{8} = \boxed{\frac{7qL^2}{8}}$$

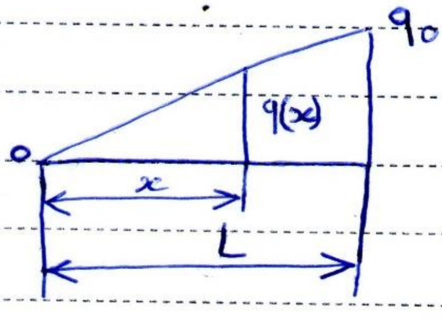
EX03 :



solution :



$\Leftrightarrow$



$$\frac{q(x)}{q_0} = \frac{x}{L} \Rightarrow \boxed{q(x) = \frac{q_0 \cdot x}{L}}$$

$$\Sigma F = 0 \Rightarrow F + \frac{q(x) \cdot x}{2} = 0 \Rightarrow \boxed{F = -\frac{q(x) \cdot x}{2}}$$

$$\boxed{F = \frac{-q_0 \cdot x^2}{2 \cdot L}} \Rightarrow \text{pour } x \Rightarrow F = 0$$

$$\text{pour } x = L \Rightarrow F = \frac{-q_0 \cdot L}{2}$$

$$\Sigma M_{/o} = 0 \Rightarrow M + q(x) \cdot x/_2 \cdot \frac{x}{3} = 0 \Rightarrow M + \frac{q_0 \cdot x^2}{2 \cdot L} \cdot \frac{x}{3} = 0$$

$$\boxed{M = -\frac{q_0 \cdot x^3}{6L}} \Rightarrow \begin{cases} x = 0 \Rightarrow M = 0 \\ x = L \Rightarrow M = -q_0 \cdot L^2/_6 \end{cases}$$