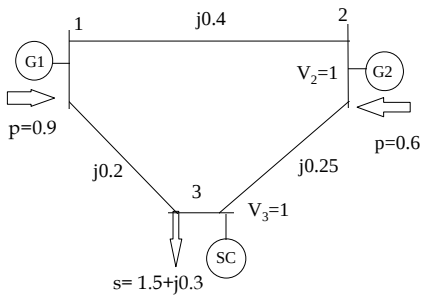
Gauss Seidel Load Flow Example

Active power demand of the 132 kV system shown in Figure is supplied by two generators G1 and G2. System voltage is supported by generator G2 and a large synchronous compensator SC, which both maintain the voltage at 1 p.u. at their respective nodes. Generator G1, connected at node 1, has no reactive power capacity available for voltage control.



- Form the Ybus matrix for this system.
- Perform two iterations of the Gauss – Seidel load flow.

Ybus matrix

$z_{12} = 0 + j0.4$ p.u $\qquad z_{13} = 0 + j0.2\,p.u \qquad z_{23} = 0 + j0.25\,p.u$

$y_{12} = \dfrac{1}{z_{12}} = -j2.5$ p.u. $\qquad y_{13} = \dfrac{1}{z_{13}} = -j5\,p.u. \qquad y_{23} = \dfrac{1}{z_{23}} = -j4\,p.u.$

$Y_{11} = y_{12} + y_{13} = -j2.5 + (-j5) = -j7.5$ p.u.

$Y_{22} = y_{12} + y_{23} = -j2.5 + (-j4) = -j6.5$ p.u.

$Y_{33} = y_{13} + y_{23} = -j5 + (-j4) = -j9$ p.u.

$Y_{12} = Y_{21} = -y_{12} = j2.5$p.u.

$Y_{13} = Y_{31} = -y_{13} = j5$p.u.

$Y_{23} = Y_{32} = -y_{23} = j4$p.u.

$$Y = j\begin{bmatrix} -7.5 & 2.5 & 5 \\ 2.5 & -6.5 & 4 \\ 5 & 4 & -9 \end{bmatrix}$$

Bus definition

- Slack bus is bus 2, as generator 1 has no reactive power capacity available for voltage control.
- PV bus is bus 3 as SC keeps voltage at 1 p.u.
- PQ bus is bus 1

Initialisation

$V_1^{(0)} = 1 + j0$ PQ bus

$V_2^{(0)} = V_2^{spec} = 1 + j0$ Slack bus

$V_3^{(0)} = V_3^{spec} = 1 + j0$ PV bus

$$Q_3^{(0)} = -\mathrm{Im}\{V_3^{(0)*} \cdot (Y_{31} \cdot V_1^{(0)} + Y_{32} \cdot V_2^{(0)} + Y_{33} \cdot V_3^{(0)})\}$$
$$= -\mathrm{Im}\{(1 - j0) \cdot [j5 \cdot (1 + j0) + j4 \cdot (1 + j0) + (-j9) \cdot (1 + j0)]\} = 0 \text{ p.u.}$$

$$S_3^{(0)} = -\mathrm{Re}(s_3) + jQ_3^{(0)} = -\mathrm{Re}(1.5 + j0.3) + j0 = -1.5 \text{ p.u}$$

Note: SC does not produce real power

First iteration

$$V_1^{(1)} = \frac{1}{Y_{11}} \cdot \left(\frac{S_1^*}{V_1^{(0)*}} - Y_{12} \cdot V_2^{(0)} - Y_{13} \cdot V_3^{(0)}\right)$$

$$= \frac{1}{-j7.5} \cdot \left(\frac{0.9 - j0}{1 - j0} - j2.5 \cdot (1 + j0) - j5 \cdot (1 + j0)\right)$$

$$= 1 + \text{j}\,0.12 \quad \text{p.u.}$$

$$\tilde{V}_3^{(1)} = \frac{1}{Y_{33}} \cdot \left(\frac{S_3^{(0)*}}{V_3^{(0)*}} - Y_{31} \cdot V_1^{(1)} - Y_{32} \cdot V_2^{(0)}\right)$$

$$= \frac{1}{-j9} \cdot \left(\frac{-1.5}{1 - j0} - j5 \cdot (1 + j0.12) - j4 \cdot (1 + j0)\right) = 1 - \text{j}0.1\ \text{p.u}$$

$$V_3^{(1)} = \left|V_3^{spec}\right| \frac{\tilde{V}_3^{(1)}}{\left|\tilde{V}_3^{(1)}\right|} = 1 \cdot \frac{1 - j0.1}{\left|1 - j0.1\right|} = 0.995 - \text{j}0.0995 \text{ p.u.}$$

First iteration

$$S_2^{(1)} = V_2^{(0)} \cdot \left(Y_{21} \cdot V_1^{(1)} + Y_{22} \cdot V_2^{(0)} + Y_{23} \cdot V_3^{(1)}\right)^*$$

$$= (1 + j0) \cdot (j2.5 \cdot (1 + j0.12) + (-6.5) \cdot (1 + j0) + j4 \cdot (0.995 - j0.0995))^*$$

$$= 0.0980 + j0.0199 \quad \text{p.u.}$$

$$Q_3^{(1)} = -\text{Im}(V_3^{(1)*} \cdot (Y_{31} \cdot V_1^{(1)} + Y_{32} \cdot V_2^{(0)} + Y_{33} \cdot V_3^{(1)})) = 0.1044\,\text{p.u.}$$

$$S_3^{(1)} = \text{Re}(S_3^{(0)}) + jQ_3^{(1)} = -1.5 + \text{j}\,0.1044 \text{ p.u}$$

Power flow and mismatches

$$S_{12} = V_1^{(1)} \cdot ((V_1^{(1)} - V_2^{(0)}) \cdot y_{12})^* = 0.3 + j0.036 \;\; p.\text{u.}$$

$$S_{21} = V_2^{(0)} \cdot ((V_2^{(0)} - V_1^{(1)}) \cdot y_{12})^* = -0.3 \text{ p.u}$$

$$S_{13} = V_1^{(1)} \cdot ((V_1^{(1)} - V_3^{(1)}) \cdot y_{13})^* = 100945 + j0.1565 \text{ p.u}$$

$$S_{31} = V_3^{(1)} \cdot ((V_3^{(1)} - V_1^{(1)}) \cdot y_{13})^* = -1.0945 + j0.0845 \text{p.u}$$

$$S_{23} = V_2^{(0)} \cdot ((V_2^{(0)} - V_3^{(1)}) \cdot y_{23})^* = 0.3980 + j0.0199 \text{ p.u}$$

$$S_{32} = V_3^{(1)} \cdot ((V_3^{(1)} - V_2^{(0)}) \cdot y_{23})^* = -0.3980 + j0.0199 \text{ p.u}$$

$$\Delta S_1^{(1)} = S_1 - S_{12} - S_{13} = -0.4945 - j0.1925 \text{ p.u}$$

$$\Delta S_2^{(1)} = S_2^{(1)} - S_{23} - S_{21} = 0 \quad \text{p.u} \quad \text{(no mismatch at slack bus)}$$

$$\Delta S_3^{(1)} = S_3^{(1)} - S_{32} - S_{31} = \text{- } 0.0074 + \text{ j0 p.u}$$

$$\Delta V_1^{(1)} = \left| V_1^{(1)} - V_1^{(0)} \right| = \left| (1 + j0.12) - (1 + j0) \right| = 0.12 \;\; p.\text{u}$$

$$\Delta V_3^{(1)} = \left| V_3^{(1)} - V_3^{(0)} \right| = \left| (0.995 - j0.995) - (1 + j0) \right| = 0.0996 \text{ p.u.}$$

Second iteration

$$V_1^{(2)} = \frac{1}{Y_{11}} \cdot \left(\frac{S_1^*}{V_1^{(1)*}} - Y_{12} \cdot V_2^{(0)} - Y_{13} \cdot V_3^{(1)} \right)$$

$$= \frac{1}{-j7.5} \cdot \left(\frac{0.9 - j0}{1 - j0.12} - j2.5 \cdot (1 + j0) - j5 \cdot (0.995 - j0.0995) \right)$$

$$= 0.9825 + j0.0520 \text{ p.u}$$

$$\tilde{V}_3^{(2)} = \frac{1}{Y_{33}} \cdot \left(\frac{S_3^{(1)*}}{V_3^{(1)'}} - Y_{31} \cdot V_1^{(2)} - Y_{32} \cdot V_2^{(0)} \right)$$

$$== \frac{1}{-j9} \cdot \left(\frac{-1.5 - j0.1044}{0.995 + j0.0995} - j5 \cdot (0.9825 + j0.0520) - j4 \cdot (1 + j0) \right)$$

$$= \text{ } 0.9852 \text{ - j}0.1381 \text{ p.u}$$

$$V_3^{(2)} = \frac{\tilde{V}_3^{(2)}}{\left| \tilde{V}_3^{(2)} \right|} = \frac{0.9852 - j0.1381}{\left| 0.9852 - j0.1381 \right|} = 0.9903 - j0.1388 \text{ p.u}$$

Second iteration

$$S_2^{(2)} = V_2^{(0)} \cdot \left(Y_{21} \cdot V_1^{(2)} + Y_{22} \cdot V_2^{(0)} + Y_{23} \cdot V_3^{(2)}\right)^*$$
$$= (1 + j0) \cdot (j2.5 \cdot (0.9825 + j0.0520) + (-j6.5) \cdot (1 + j0) + j4 \cdot (0.995 - j0.0995))$$
$$= 0.4255 + j0.0825 \text{ p.u.}$$

$$Q_3^{(2)} = -\text{Im}(V_3^{(2)*} \cdot (Y_{31} \cdot V_1^{(2)} + Y_{32} \cdot V_2^{(0)} + Y_{33} \cdot V_3^{(2)})) = 0.2099 \text{ p.u.}$$
$$S_3^{(2)} = \text{Re}(S_3^{(1)}) + jQ_3^{(2)} = -1.5 + \text{j}0.2099 \text{ p.u}$$

$$S_{sc} = S_3^{(2)} + s_3 = (-1.5 + \text{j}0.2099) + (1.5 + \text{j}0.3) = \text{j}0.5099 \text{ p.u}$$

Power flow and mismatches

$$S_{12} = V_1^{(2)} \cdot ((V_1^{(2)} - V_2^{(0)}) \cdot y_{12})^* = -0.1299 + j0.0438 \text{ p.u.}$$
$$S_{21} = V_2^{(0)} \cdot ((V_2^{(0)} - V_1^{(2)}) \cdot y_{12})^* = -0.1299 + j0.0438 \text{ p.u.}$$
$$S_{13} = V_1^{(2)} \cdot ((V_1^{(2)} - V_3^{(2)}) \cdot y_{13})^* = 0.939 + 0.0112 \text{ p.u}$$
$$S_{31} = V_3^{(2)} \cdot ((V_3^{(2)} - V_1^{(2)}) \cdot y_{13})^* = -0.9393 + 0.1712 \text{ p.u}$$
$$S_{23} = V_2^{(0)} \cdot ((V_2^{(0)} - V_3^{(2)}) \cdot y_{23})^* = 0.5554 + \text{j}0.03871 \text{ p.u}$$
$$S_{32} = V_3^{(2)} \cdot ((V_3^{(2)} - V_2^{(0)}) \cdot y_{23})^* = -0.5554 + \text{j}0.03871 \text{ p.u}$$
$$\Delta S_1^{(2)} = S_1 - S_{12} - S_{13} = -0.1692 + \text{j}0.0251 \text{ p.u}$$
$$\Delta S_2^{(2)} = S_2^{(2)} - S_{23} - S_{21} = 0 \text{ p.u}$$
$$\Delta S_3^{(2)} = S_3^{(2)} - S_{31} - S_{32} = -0.0053 \text{ p.u}$$

$$\Delta V_1^{(2)} = \left|V_1^{(2)} - V_1^{(1)}\right| = \left|(0.9825 + j0.0520) - (1 + j0.12)\right| = 0.0703 \text{ p.u}$$

$$\Delta V_3^{(2)} = \left|V_3^{(2)} - V_3^{(1)}\right| = \left|(0.9903 - j0.1388) - (0.995 - j0.0995)\right| = 0.0396 \text{ p.u.}$$

Inclusion of PV Buses in G-S

- To solve for V_i at PV bus we must first make a guess of Q_i:

$$S_i^* = V_i^* \sum_{k=1}^{n} Y_{ik} V_k = P_i - jQ_i$$

- Hence

$$Q_i^{(\nu)} = -\mathrm{Im}\left[V_i^{(\nu)*} \sum_{k=1}^{n} Y_{ik} V_k^{(\nu)} \right]$$

- In the iteration we use

$$S_i^{(\nu)} = P_i + jQ_i^{(\nu)}$$

Inclusion of PV Buses

Tentatively solve for $V_i^{(\nu+1)}$

$$\tilde{V}_i^{(\nu+1)} = \frac{1}{Y_{ii}} \left(\frac{S_i^{(\nu)*}}{V_i^{(\nu)*}} - \sum_{k=1, k\neq i}^{n} Y_{ik} V_k^{(\nu)} \right)$$

But since $|V_i|$ is specified, replace $\left|\tilde{V}_i^{(\nu+1)}\right|$ by $|V_i|$